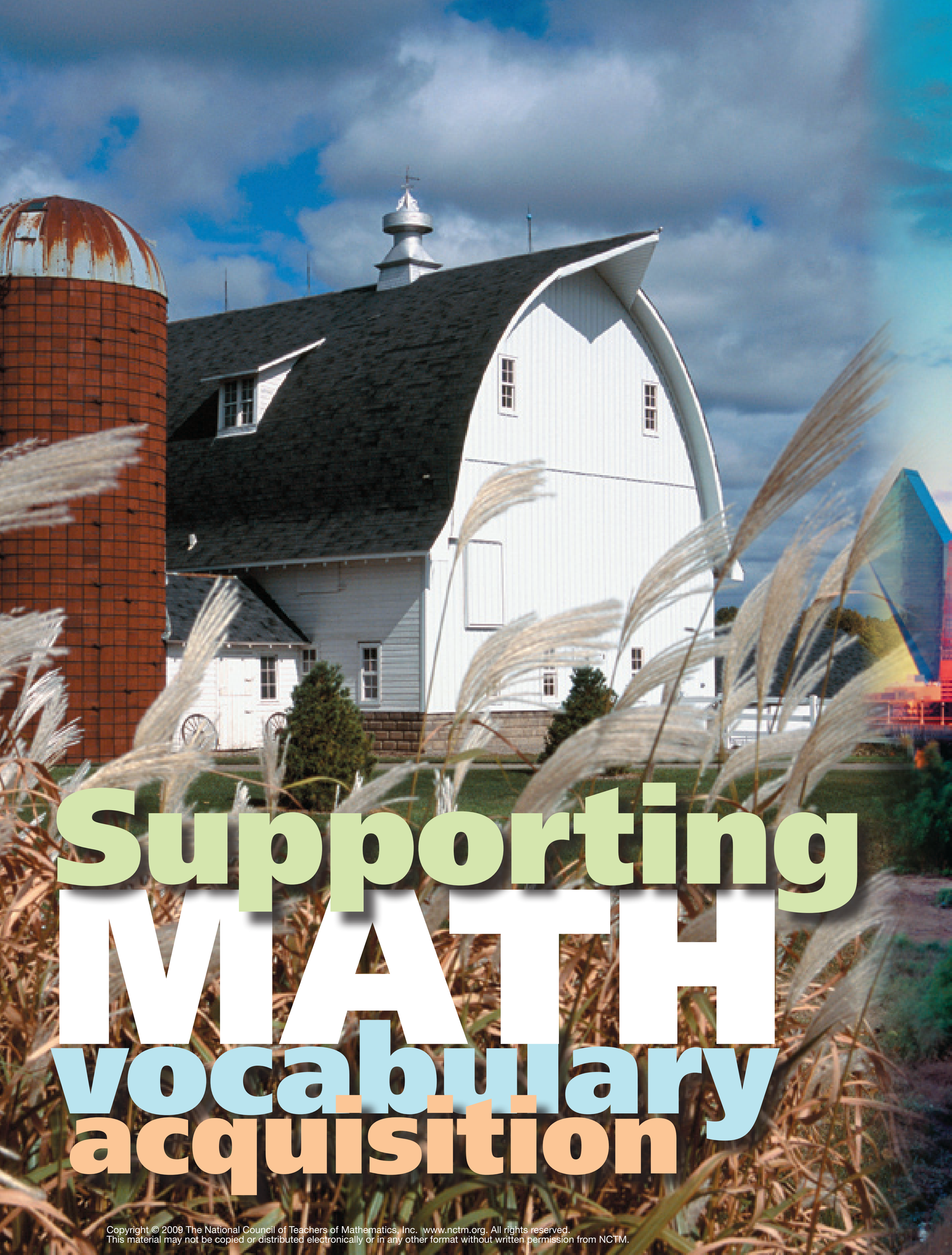


Supporting MATH vocabulary acquisition

Learn *when* and *how* teachers can use rich mathematical vocabulary to develop and maximize students' learning, particularly English Language Learners and struggling readers.

By Jennifer M. Bay-Williams and Stefanie Livers

Providing appropriate language support is important for all students and essential to the success of English Language Learners (ELLs), struggling readers, and students with learning disabilities. In a mathematics classroom, the support includes the ongoing development of everyday vocabulary—such as names of fruit and actions such as skipping—and academic vocabulary, such as *perimeter*, *product*, and *line of best fit*. The importance of explicitly teaching vocabulary has been well documented and has received much attention (see, for example, Echevarria, Vogt, and Short 2007; Marzano and Pickering 2005), yet *when* and *how* to introduce the vocabulary has received far less attention. We suggest that this timing is essential to the development of both language and mathematical understanding.

In Standards-based lessons (those that reflect the National Council of Teachers of Mathematics (NCTM) Teaching and Learning Principles and the NCTM Process Standards of Problem Solving, Reasoning, Communication, Representations, and Connections), students must apply their prior knowledge to explore and understand new concepts. A Standards-based approach to teaching and learning means an increased presence and use of language. Although it may be tempting to remove the language from the task as an accommodation, the language may be providing the very context or background knowledge to help the learner make sense of the mathematics content (Echevarria, Vogt, and Short 2007).

On the other hand, a mathematics lesson with too much new vocabulary can prevent a student from exploring mathematical ideas. Below we share considerations and strategies to guide decision making related to when and how to provide appropriate vocabulary support to maximize a student's access to mathematics.

The dilemma

The essence of the Equity Principle (NCTM 2000) is the goal of maintaining high expectations while providing appropriate adaptations to the learning environment. The phrase comprehensible input is used to describe the process of making lesson content accessible to English Language Learners (ELLs). One crucial consideration in making the content comprehensible is intentional vocabulary support—helping students with the academic or technical language as well as the everyday language that the lesson involves. But what is appropriate vocabulary support?

A common suggestion is to review known vocabulary and preview new vocabulary for each lesson. Opening a lesson in this manner can provide students with the verbal tools to participate effectively in the lesson. However, taking time for vocabulary at the start of the lesson means less time for exploration during the lesson. In particular, previewing vocabulary can quickly become previewing content, which requires additional time. Soon a third of the instructional time is gone. Moreover, previewing the content can inadvertently lower the level of cognitive demand by showing students what they would otherwise be figuring out during the lesson.

On the other hand, launching into a lesson without any attention to the vocabulary can leave students without the support they need to understand and engage in the problems or activities. Students can lose much time looking up word meanings and making false starts to the problem because they lack knowledge of the context, not the content.

What is the solution to this dilemma? As with most considerations in teaching, the answer depends on various factors, including students' prior knowledge, language skills, and interests. The focus of our discussion targets *context* and *content* considerations, in particular, strategies for when the content or context are *review* or *new*. Differentiating our thinking about vocabulary support in this way helps us make good choices about supporting the needs of all students. Identifying where vocabulary falls within the four categories described in **table 1** makes a difference in *when* and *how* to provide vocabulary support.

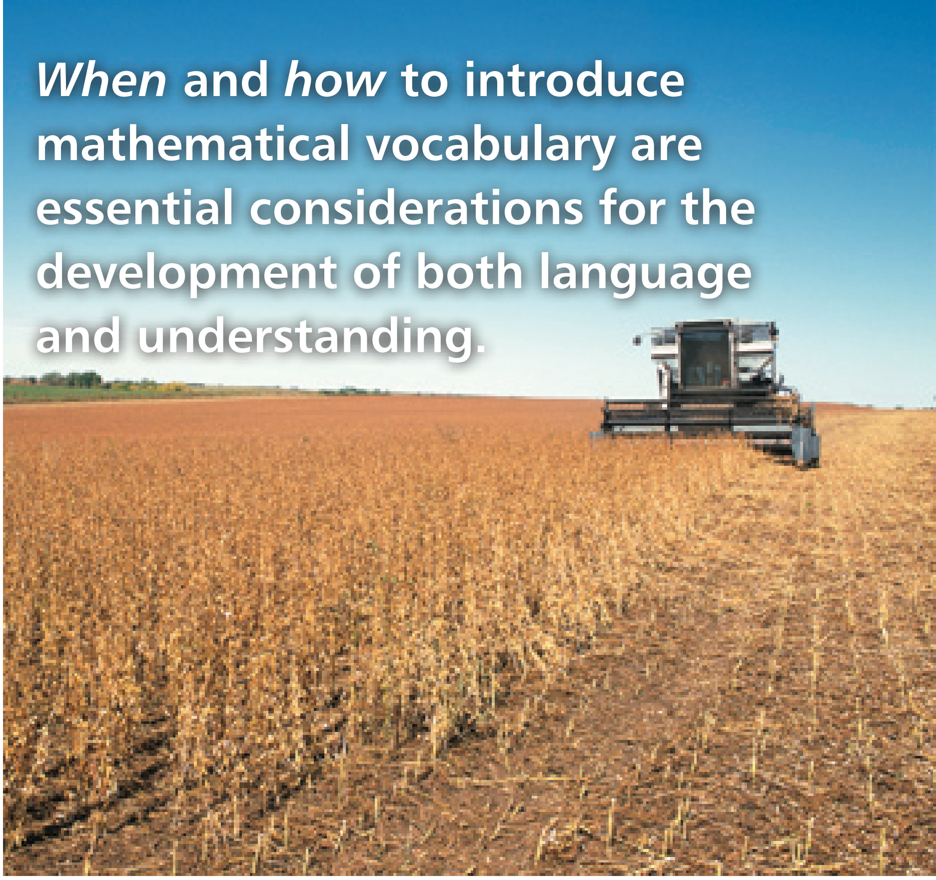
TABLE 1

Analysis tools allow teachers to characterize a lesson's vocabulary.

	Mathematics-related	Context-related
New		
Review		

Context

As students explore mathematics through a meaningful context, they are able to derive meaning of abstract mathematics concepts and see that mathematics is an integral part of their world. Although context *can* support learning, the connection between context and a mathematical idea is not inherent in the mathematics and must be carefully developed by the teacher. The following context considerations can guide decision making about when and how to support vocabulary development to enhance mathematics learning. The questions that follow are intentionally ordered in the spirit of a flow chart of reflective questions to make decisions about context-related vocabulary.



Is the context culturally relevant?

Cultural relevancy means that students can relate to the topic and become engaged in the problem because they understand its context and find it interesting. Culturally relevant contexts are not always obvious. Consider, for example, a group of multinational students in an urban setting who are exploring fractions in the context of farming. Will the farmer's plan to devote portions of his land to various crops serve as a relevant context for these students to learn about fractions? Students may have experiences in farming and understand the concept of wanting to select different crops to ensure a profitable harvest, in which case the problem *is* a culturally relevant example. However, this context may have little interest for students or may be completely unfamiliar, in which case the context will be not be serving its purpose of supporting student thinking about the mathematical ideas. If the context is not culturally relevant, then replace the context. In this case, the context could be changed to diagramming space within the classroom for various purposes (e.g., reading area, discussion circle, desks, and resources).

Is the context familiar or new?

Once the context is determined to be culturally relevant, the next consideration relates to how familiar the vocabulary will be for students. If the context is familiar and vocabulary is known, then it is a good context for exploring the mathematics and adding new vocabulary as needed. If the context is new, then consideration must be given to the importance of learning that context. If the context is not a priority for the students to know (and will not reappear in later lessons) replace the context. Even if the context is culturally relevant, it may include too much new language for students to learn, distracting from the lesson goals. If the context is one that students must be able to understand and verbalize, then consider keeping the context and giving careful attention to the needed vocabulary support.

In the previous example, for instance, farming may be culturally relevant for students, but ELLs may not have learned farming terminology, such as *acre*, *plot*, *crop*, *field*, *land*, *corn*, *wheat*, *soybean*, and *hay*. If the context is instead about something familiar, such as a classroom, the "linguistic load" of the farming example is removed, and students are more likely to be able to participate in the discourse around the mathematics concepts.

Another option is to keep the context of farming but adapt the language within the problem to reduce the linguistic load. The context vocabulary can be limited to *land* and a short list of crops, noting each one on the board or a poster with a picture near it and a translation to help students remember and use the needed context-related vocabulary.

Can the context be reused?

Selecting context can take advantage of both the meaning and vocabulary that students are developing in other content areas. For example, if students are studying maps in social studies, the context of a fraction lesson can shift to regions on a map. If students are exploring species in science class, a fraction lesson can be developed to look at populations of species or even fractions of the regions where the species live. This interdisciplinary approach further addresses the linguistic load a student encounters in a school day.

When do I provide support?

Returning to **table 1**, if the context is new—and you have decided it is appropriate to keep—then your planning must allot time for an extended vocabulary preview at the start of the lesson. This can include such items as regalia (real objects used in the problem), visuals, sketches, notations on the board, or a handout with vocabulary support. Also include choral responses and word repetition. Emphasize the importance of using these terms during the lesson and build in opportunities for sharing in which students must use the context-related vocabulary. Using sentence starters, pair-shares, and graphic organizers increases the opportunities for students to use the context-related vocabulary, while simultaneously supporting the lesson's content goals.

When the lesson's context-related vocabulary is a review, incorporate engaging ways to review at the start of the lesson. Many of these activities can put the context vocabulary in the mouths of learners in less than five minutes, protecting the lesson time for exploring mathematics. (See seven of our favorite activities appended to the online version of this article as an appendix at **www.nctm.org/tcm**.)

Content

In a Standards-based learning environment, students must be able to communicate mathematical ideas. The overarching consideration in providing mathematics vocabulary support is how the inclusion or removal of that terminology impacts the students' access to important and challenging mathematics. Ivory, Caparro, and Ball (2003) write that attempts to limit a lesson's vocabulary and language use, although intended to support students with limited English proficiency, can instead hinder their learning because students are not engaged in verbalizing their thinking. In this section, we discuss considerations when planning for content-related vocabulary support.

Is the mathematics vocabulary new?

Read the vignette in **figure 1**, a dialogue focused on introducing perimeter. Has the opportunity to engage in challenging mathematics been enhanced or diminished?

The teacher in this vignette is modeling effective practices in previewing vocabulary by using visuals, choral response, and repetition. Mathematically, she is focused on the lesson's key words. However, her vocabulary preview steps over the line and addresses how to do the mathematics. (She presents an algorithm for finding perimeter.) Moreover, the focus on the mathematics does not connect to a student's real experiences.

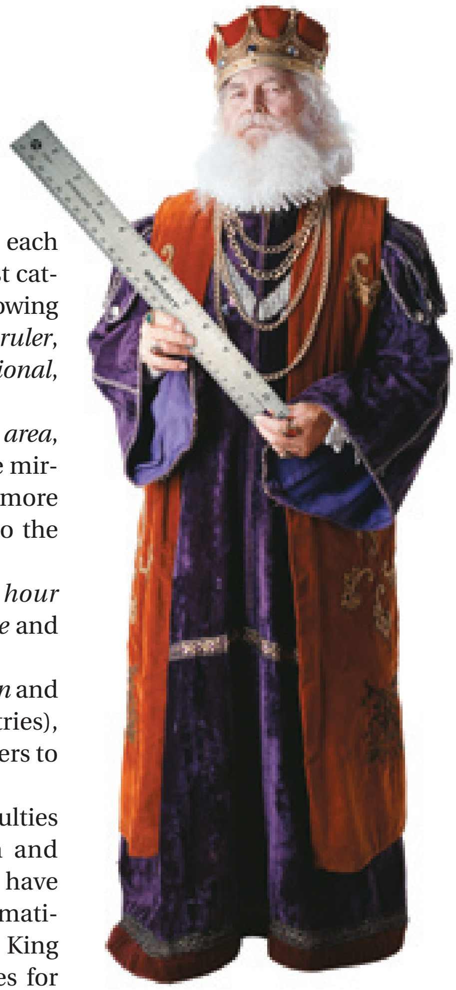
Slight shifts in the teacher's preview, however, could address the issues above and better support students' access to both vocabulary and mathematics. First, consider replacing the first teacher move (pointing at a rectangle) with a rectangle in a context, such as an overhead sketch of a fenced garden or the border of a photograph. Second, consider shifting the focus of the preview from "adding the sides" to "finding the measure," focusing on how *long* the perimeter is, and using string or a meter stick as a visual aid. These phrases focus on the concept of perimeter, provide key terminology needed for the lesson, and avoid telling students how to find a rectangle's perimeter.

With new content, key words may need to be previewed (e.g., *sides*, *measure*, and *long*) so that students can use appropriate terminology in the lesson; but other times, the new vocabulary should *not* be introduced until after the concept has been explored (Garrison and Mora 2003). An introductory lesson in multiplication may preview language such as *group*, an everyday vocabulary word, but save the discus-

FIGURE 1

Slight shifts on the teacher's part (**T**) could have better supported students' (**S**) access to both vocabulary and math.

T: [*Pointing at a rectangle on the overhead projector*] What is the name of this shape?
S1: Rectangle.
T: Today we are exploring the perimeter [*pointing around the sides*] of rectangles, the distance around the outside. Say, "perimeter."
Class: Perimeter.
T: And, what is the perimeter?
S2: The part around … the lines.
T: Can you tell me what the lines are called?
S3: Sides?
T: Sides; say, "sides."
Class: Sides.
T: We find the *perimeter* by adding the *sides*. Count with me how many units we have along each side. [*The class counts units on each side. The teacher writes, "3 + 5 + 3 + 5 = 16 inches."*]
T: [*Pointing at 16*] What is this called?
Class: Perimeter.
T: [*Pointing at addends*] What are these numbers?
Class: Sides.
T: On the papers I am handing out [*distributes papers*], you will find the perimeter of each rectangle by finding the measures of each side.



sion of the most important word, *multiply*, and the related term *multiplication* for the post-exploration discussion. The discussion might progress like the dialogue in the vignette about a postlesson discussion on multiplication **(see fig. 2)**.

Because multiplication ideas are developed in this lesson, previewing the vocabulary does not make sense; students lack the conceptual foundation for what the vocabulary means. A postlesson discussion is necessary to simultaneously support vocabulary development *and* conceptual understanding of the math concepts. Notice also that the students will have more opportunities to use their new vocabulary in the subsequent examples (as noted by the teacher).

How did the perimeter and multiplication lessons differ? In the perimeter example, it was possible to preexpose students to the key word *perimeter* without having to explain the content to be learned in the lesson. In fact, having the words *perimeter* and *measure* enabled students to use the academic vocabulary as they measured and explored rectangles. In the multiplication example, there was no way to preview the terms *multiply* or *times* without explaining the content, so the explicit vocabulary development was better suited to a postlesson discussion.

Does the vocabulary have multiple meanings?

In the multiplication lesson, the teacher took extra time to ask students where they had heard or used the word *times*. She was aware that the word *times* was probably familiar to students but that the meaning within the context of multiplication was different. Words used in mathematics often take on different meanings from when they are used in everyday life, posing a challenge for all students but particularly for ELLs. Academic mathematics vocabulary may have the following characteristics:

- **Different** use in everyday language
- **Related** but not as specific use in everyday language
- **A homonym** but not used in everyday language
- **Different** meaning from the word that translates from a student's native language

Try to think of a few words that fit into each of the previous categories. A list for the first category could include six or eight of the following words: *right*, *table*, *row*, *column*, *pattern*, *ruler*, *product*, *bar*, *odd*, *reflect* (on an idea), *rational*, *property*, *median*, *figure*, and *mean*.

The second list might include *similar*, *area*, *multiply*, *difference*, *average*, *reflect* (in the mirror), *base*, and *cube*. Verbs that take on more precise meanings in mathematics fall into the second category: *find*, *simplify*, and *show*.

Homonyms include *sum* and *some*, *hour* and *our*, *plain* and *plane*, *ad* and *add*, *rose* and *rows*.

In the last category are the words *million* and *billion* (which are reversed in some countries), and *figure* in Spanish (*la figura*), which refers to tables and graphs but not to numerals.

For a list of additional potential difficulties in learning vocabulary, see Rubenstein and Thompson (2002). For a list of words that have everyday meanings different from mathematical meanings, see Adams, Thangata, and King (2005). Both of these are excellent articles for

FIGURE 2

This vignette portrays a postlesson discussion on multiplication between a teacher (**T**) and students (**S**).

T: How did you find how many crayons were in three boxes?

S1: I added a 6 for each box of crayons—six plus six plus six—to get eighteen.

T: [*Records "$3 \times = 6 + 6 + 6 = 3 \times 6 = 18$."*] I have written what Ethan told me in two different ways. Can someone explain this second way?

S2: Three groups of six, three sixes.

T: When we have equal groups, like the three equal groups here, we can either add the number three times, or we can *multiply* by three. Say, "multiply" [*writing* multiply *on the board*].

Class: Multiply.

T: [*Pointing at the equation, the teacher says the following, first alone, then with students*] We *multiplied* the six crayons by three. We can also say that we have [*pointing*] three times six, which means we have six boxes of crayons three times.

T: Have you heard the word *times* [when you are] doing something different from math?

S3: Telling time.

S4: Telling stories, like the time I went camping.

S5: Time-out, like when you are in trouble or in sports.

T: Great ideas. So, *times* can mean several things. One thing it means in math is that we are adding the same amount, like three times six or having six three times. In the next problems that you solve, you will explain using addition *and* using our new words, *multiply* and *times*.

realizing the complexities of mathematics vocabulary. Perkins and Flores (2002) provides a rich collection of differences in Mexican and American terminology, symbolic representations, and algorithms.

Is the mathematics vocabulary a review?

Nearly all mathematics lessons include concepts that students have previously learned. This prior knowledge and its related vocabulary are essential to building new mathematical concepts.

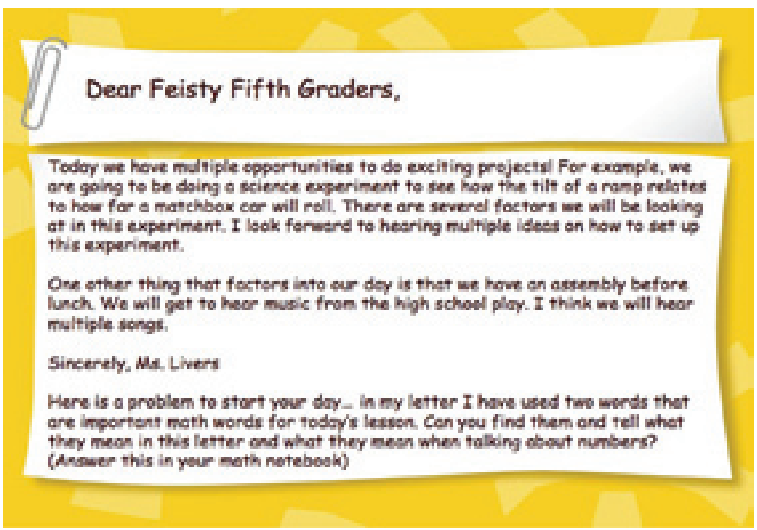
Review earlier. As already noted, an important consideration is how to review vocabulary while protecting the actual lesson time. One way to do so is to review earlier in the day rather than at the start of the lesson. Here are strategies for previewing vocabulary before the lesson:

- **A question of the day** discussed first each morning provides a nice interactive review and helps to preexpose students to vocabulary used later in the day. In grades 1–2, this question may be a data collection t-chart where students place their name clip on the appropriate answer. For example, to preview a lesson on extending patterns, provide a pattern and ask, "What kind of pattern is this?" with the column choices of "repeating pattern" and "growing pattern." Reading and responding to the question activates prior knowledge of what repeating and growing patterns are while giving the teacher insights about students' readiness for the lesson.
- **A morning message**, just like the question of the day, is a routine to begin the day that can preexpose students to vocabulary they will need in the mathematics lesson. Letters may be conversational or may pose problems to students. Notice the creative fifth-grade example (see **fig. 3**) where the teacher has not only identified key review vocabulary but also focused on the fact that these words take on specific meanings in the upcoming lesson that are not necessarily how these words are used in conversation.
- **Homework prompts** are an excellent opportunity to review content with students. Although homework is usually considered a way to reinforce a finished lesson, it can be used to preview content. A teacher might have students do a scavenger hunt through magazines and newspapers for graphs, charts, or diagrams as a way of previewing and providing visuals for understanding these terms.
- **Read-alouds** can be incorporated into any time of the day. Reading children's literature that is related to a mathematics concept is an excellent way to begin a lesson and to review vocabulary. Reading *The Greedy Triangle*, (Burns 1995), for example, before a lesson on polygons provides students with an opportunity to review polygon names. The repeated phrase in the story, "adding one more side and one more angle," can help students connect the type of polygon (e.g., a pentagon) with the number of sides (e.g., 5) and note the connection of the word's prefix (e.g., *penta*-) to other terms that use this prefix.

Review during the lesson. When previously learned mathematics vocabulary and related concepts are required for a lesson, quick and fun reviews place those essential words back in the minds and mouths of students. In conserving time for inquiry within the lesson, such review can and should take less than

FIGURE 3

Preview key vocabulary and make explicit the difference between everyday meanings and mathematical meanings.


Dear Feisty Fifth Graders,

Today we have multiple opportunities to do exciting projects! For example, we are going to be doing a science experiment to see how the tilt of a ramp relates to how far a matchbox car will roll. There are several factors we will be looking at in this experiment. I look forward to hearing multiple ideas on how to set up this experiment.

One other thing that factors into our day is that we have an assembly before lunch. We will get to hear music from the high school play. I think we will hear multiple songs.

Sincerely, Ms. Livers

Here is a problem to start your day... In my letter I have used two words that are important math words for today's lesson. Can you find them and tell what they mean in this letter and what they mean when talking about numbers? (Answer this in your math notebook) five minutes at the start of class, with essential words left in sight for use throughout the lesson. Activities can be used over and over again and, once learned, take little time to implement. In addition to starting a lesson, motivating, well-received activities can be used as "sponges" (activities that "soak up" those extra minutes when students are waiting to transition to another lesson or teacher) or as a full lesson when the lesson goal is to reinforce essential vocabulary. Finally, such activities are good family mathematics activities, extending students' opportunities to practice vocabulary. (Again, seven of our favorites are appended to the online version of this article at **www.nctm.org/tcm**.)

Summary

Two complexities exist in making decisions about vocabulary support. One is to navigate between offering explicit vocabulary instruction while maintaining the integrity of a Standards-based mathematics lesson. A second is to think about vocabulary differently, on the basis of whether it is new or a review and whether it is context related (everyday language) or mathematics related (academic). In analyzing lessons and vocabulary in these ways, teachers can make decisions about how and when to emphasize vocabulary to best support student learning of language and mathematics. Explicit vocabulary instruction and challenging, Standards-based mathematics can—must—be the norm for classroom instruction if all students are to successfully learn mathematics.

BIBLIOGRAPHY

Adams, Thomasena L., Fiona Thangata, and Cindy King. "Weigh to Go! Exploring Mathematical Language." *Mathematics Teaching in the Middle School* 10 (May 2005): 444–48.

Burns, Marilyn, *The Greedy Triangle*. New York: Scholastic, 1995.

Campbell, Bruce. "Differentiate Your Instruction Now!" Paper presented at the Midwest Conference on Differentiated Instruction, Columbus, Ohio, July 2005.

Echevarria, Jana, Mary Ellen Vogt, and Deborah J. Short. *Making Content Comprehensible for English Learners: The SIOP Model*. 3rd ed. New York: Pearson Learning, 2007.

Garrison, Leslie, and Jill K. Mora. "Adapting Mathematics Instruction for English-Language Learners: The Language-Concept Connection." In *Changing the Faces of Mathematics: Perspectives on Latinos,* pp. 35-47. Reston, VA: National Council of Teachers of Mathematics, 2003.

Ivory, Gary, Dolores Caparro, and Stanley Ball. "Staff Development to Foster Latino Students' Success in Mathematics: Insights from Constructivism." In *Changing the Faces of Mathematics: Perspectives on Latinos,* pp. 113–22. Reston, VA: National Council of Teachers of Mathematics, 2003.

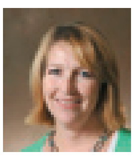
Kenney, Joan, Euthecia Hancewicz, Loretta Heuer, Diana Metsisto, and Cynthia L. Tuttle. *Literacy Strategies for Improving Mathematics Instruction.* Alexandria, VA: Association for Supervision and Curriculum Development, 2005

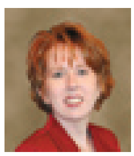
Marzano, Robert J. and Debra J. Pickering. *Building Academic Vocabulary: Teacher's Manual.* Alexandria, VA: Association for Supervision and Curriculum Development, 2005.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

Perkins, Isabel, and Alfinio Flores. "Mathematical Notations and Procedures of Recent Immigrant Students." *Mathematics Teaching in the Middle School* 7 (February 2002): 346–51.

Rubenstein, Rheta, and Denisse Thompson. "Understanding and Supporting Children's Mathematical Vocabulary Development." *Teaching Children Mathematics* 9 (October 2002): 107–12.





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Activity suggestions for a math vocabulary review are appended to the online version of this article at **www.nctm.org/tcm**.

"Supporting math vocabulary acquisition" menu activities

Mathematics vocabulary review

Activities in the vocabulary review can be used repeatedly and are good family mathematics activities.

Game Title	Description
10,000 Pyramid	This game is played just like the classic television show. The teacher creates a pyramid filled with six review words (three on the bottom, two in the middle, and one on top). The pyramid can be on a PowerPoint slide, transparency, or poster. Student pairs face each other; only one student can see the pyramid. Starting at the bottom and working up through the terms, this person describes each word, trying to get his or her partner to say the word. Drawing pictures can replace using verbal explanations (adapted from Marzano and Pickering 2005).
I Have, Who Has	In this round robin game, each student has a card with a key word and the definition of some other key word. Students listen for the definition of the word that they have, and then they read the next definition. The first student reads only the bottom of his or her card. For example, "Who has a polygon with four equal sides and four right angles?" A student who has the word *square* on the top of the card would say, "I have a square," and proceed to read the definition written on the bottom of his or her card. Then another student jumps in: "I have … " To create the cards, make a two-column table. List the words in the left column. Add the definitions for each word in the right column but in the previous row. For example, the definition for the third word would be placed in the second row. The definition for the word in the second row would be in the first row. Each row becomes a card for the game. See Kenney et al. 2005 and Rubenstein and Thompson 2002 for more ideas for this game. Already-created sets can be found through a Web search. Once made, these decks can be used over and over again.
Heads Up	Attach mathematical vocabulary word cards to headbands. As students enter class, place a headband on their forehead. (They cannot see their word.) When the signal is given, students must go around the room asking one another three questions that will help them figure out the word on their headband. A variation is to place a number on the headbands and have students ask questions using factors or multiples (e.g., Is my number a multiple of 5? Is it less than 30?).
Matching Puzzle Cards	Students or the teacher can create these cards before the lesson. A word is written at the top of a note card, and the definition is written at the bottom. Different kinds of cuts (zigzags, waves, squiggles) are made to separate the word from the definition. The full set is mixed up and placed on the table for students to match the word with its definition (Campbell 2005).
Sequence Cards	Distribute cards to individual students or to pairs of students. Students must come to the front of the room and position themselves in sequential order. For example, the multiples of 4 could be distributed, and students would get in sequential order. Large numbers can be used, and students can practice making comparison statements (e.g., 3451 is less than 4051), reinforcing both comparison language (*greater than*, *less than*) and how to read multidigit numbers. Another example would be to list the types of polygons on cards and have students get in sequential order according to the number of sides for each polygon (Campbell 2005).
Two-Dice Challenge	Working in pairs or small groups, students roll two vocabulary dice. One die has the following words, one on each face: *draw*, *define*, *describe*, *synonyms*, *antonyms*, and *examples*. The other die has six review words. Student groups roll the pair of dice and do the task stated on the one die for the word on the other die.
BOOGLE	Have students fold a piece of paper in half. Give them two minutes to write everything they know and remember about a vocabulary word on the left half of the paper. When time is up, have students share with the others at their table for another minute and record anything they missed on the right side of their folded paper. Each student partners with someone from another table to see if they left out anything else, which they add to the right column as well.