STUDENT
ACHIEVEMENT
PARTNERS

Instructional Practice Toolkit

Mathematics – Grade 3

Facilitator Resources

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Facilitator Guide: Instructional Practice Toolkit (Mathematics)

Purpose and Audience

The Instructional Practice Toolkit (IPT or Toolkit) is designed for use by coaches and instructional leaders to help teachers, and those who support teachers, build understanding and experience with College and Career Readiness (CCR) standards-aligned instruction. It is designed to highlight the throughline from designing and planning a lesson, to teaching it, and finally to analyzing student work to see if the intended outcomes were achieved for students. Learning how to recognize and support effective teaching and learning practices that reflect the specific Shifts of CCR standards helps to develop shared, complementary expertise across districts, schools, and classrooms.

The IPT is designed for educators with varying levels of experience with the CCR standards and the Shifts. However, the IPT requires the facilitator and participants to have basic knowledge of the Instructional Shifts required by the standards as well as familiarity with the Instructional Practice Guide (IPG). Throughout the IPT, there are recommendations for resources and additional training to build the capacity of all learners in key content areas. Facilitators should be aware of the capacity and goals of the participants and adjust the content and the pace of learning to meet the specific needs of the audience.

The three Shifts in instruction for mathematics are:

Focus strongly where the standards focus.

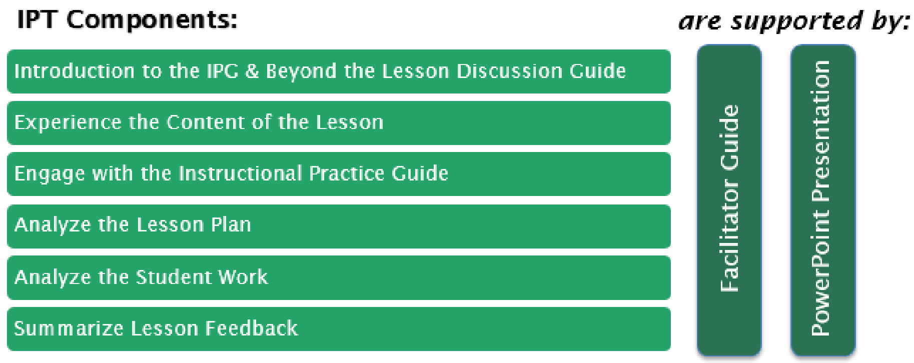
Coherence: Think across grades and link to major topics within grades.

Rigor: In major topics pursue conceptual understanding, procedural skill and fluency, and application with equal intensity.

Learning Goals

- Examine and discuss evidence of standards-aligned practice using content-specific tools and resources including Instructional Practice Guide and Beyond the Lesson Discussion Guide
- Engage with authentic lesson content and discuss the related Shifts and specific standards required (e.g., do the math problem)
- Observe lesson video and gather evidence of teacher and student actions that exemplify standards-aligned instruction
- Analyze and interpret lesson plans and student work to collect and discuss evidence of standards-aligned practice
- Summarize overall trends of standards-aligned practice and discuss implications and next steps based on a variety of specific roles and context

Overview of the Instructional Practice Toolkit



The Instructional Practice Toolkit is anchored in the Core Actions of the Instructional Practice Guide. In addition to observing a lesson using the IPG and reflecting on the Beyond the Lesson: Discussion Guide (BTL) questions, participants will analyze the lesson plan and the student work associated with that lesson. The provided Feedback Summary form will be used at the end of the IPT to summarize feedback. It will highlight both the lesson's strengths and opportunities for improvement specifically against the Core Actions and their indicators. A PowerPoint presentation is provided to guide the learning and activities throughout the IPT and serves as the anchor for delivering the material.

How to Facilitate the IPT

To prepare to facilitate the IPT, first read through the entire PowerPoint including the notes section on each slide. The notes sections detail key talking points, instructions for activities, and resources for providing additional background knowledge for participants as needed. In addition, before delivering the IPT, facilitators should complete each of the activities that the participants will be assigned. Model responses are provided for the facilitator to reference as they prepare for the session.

It is recommended that participants be organized into groups small enough to promote evidence-based discussion and participation from every member of the group, but large enough to allow for varied opinions. Four to eight people per group is ideal.

Timeframe to Complete all Components of the Toolkit: 6 - 8 hours

The Toolkit could be delivered:

- In a one-day professional learning session
- Broken into shorter sessions as part of an extended professional development learning opportunity or PLC

Facilitators should be aware of the capacity and goals of the participants and adjust the content and the pace of learning to meet the specific needs of the audience.

The information in the table below can be adapted to be used with any video and associated materials.

Components and Activities	Time	Materials Needed
Introduction ▪ *Essential Question* ▪ *Learning Goals* ▪ *The Teaching and Learning Cycle* ▪ *Overview of IPT Components* ▪ *Norms*	10 - 15 minutes	For the facilitator: ▪ PowerPoint
Introduction to the Instructional Practice Guide & Beyond the Lesson Discussion Guide ▪ *The Shifts* ▪ *IPG Design* ▪ *Beyond the Lesson Guide*	30 minutes: For participants with prior knowledge of the Shifts and the IPG.	For each participant: ▪ College- and Career-Ready Shifts in Mathematics ▪ IPG (for the grade level featured, K-8 or HS) and Beyond the Lesson Guide, Mathematics
Experience the Content of the Lesson ▪ *Core Action 1* ▪ *Do the Math* ▪ *Reflection*	45 - 60 minutes	For each participant: ▪ Student Assignment - Participant Handout ▪ Standards document for the grade level featured (CCSS linked here) or your state's corresponding CCR standards ▪ Focus document (for grade level featured, K-8 only) For the facilitator: ▪ Experience the Content of the Lesson: Discussion Guide

Components and Activities	Time	Materials Needed
Engage with the Instructional Practice Guide ▪ *Core Actions 2 & 3* ▪ *Low Inference Notes* ▪ *Watch the Lesson Video* ▪ *Complete the IPG* ▪ *Beyond the Lesson Guide* ▪ *Reflection*	90 minutes If calibration is a goal, an additional 30 - 45 minutes will be required for norming discussion.	For each participant: ▪ The Observation and Feedback Cycle: Best Practices for Low Inference Notes For the facilitator: ▪ Edited video of a lesson & optional transcript ▪ IPG - Model Response ▪ Optional: Tool for capturing participant evidence (*note: must be prepared ahead of time*)
Analyze the Lesson Plan ▪ *Lesson Plan Analysis* ▪ *Reflection*	45 - 60 minutes	For each participant: ▪ Teacher-created Lesson Plan ▪ Lesson Plan Analysis - Participant Handout For the facilitator: ▪ Lesson Plan Analysis - Model Response
Analyze the Student Work ▪ *Student Work Analysis* ▪ *Reflection*	45 - 60 minutes	For each participant: ▪ Student Work Samples ▪ Student Work Analysis - Participant Handout For the facilitator: ▪ Student Work Analysis - Model Response
Summarize Lesson Feedback ▪ *The Teaching and Learning cycle* ▪ *Beyond the Lesson Guide* ▪ *Synthesize evidence from the IPG, Lesson Plan Analysis, and Student Work Analysis* ▪ *Reflection*	45 minutes	For each participant: ▪ Previously Completed Participant Handouts ▪ Feedback Summary - Participant Handout For the facilitator: ▪ Feedback Summary - Model Response

Shifts at a Glance

College- and Career-Ready Shifts in Mathematics

Focus strongly where the standards focus.

Focus: The Common Core and other college- and career-ready (CCR) standards call for a greater focus in mathematics. Rather than racing to cover topics in a mile-wide, inch-deep curriculum, CCR standards require us to significantly narrow and deepen the way time and energy the way time and energy are spent in the math classroom. We focus deeply on the Major Work* of each grade so that students can gain strong foundations: solid conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the math they know to solve problems inside and outside the math classroom.

Coherence: Think across grades and link to major topics within grades.

Thinking across grades: College- and career-ready standards are designed around coherent progressions from grade to grade. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.

Linking to major topics: Instead of allowing additional or supporting topics to detract from the focus of the grade, these concepts serve the grade-level focus. For example, instead of data displays as an end in themselves, they are an opportunity to do grade-level word problems.

Rigor: In major topics*, pursue conceptual understanding, procedural skill and fluency, and application with equal intensity.

Conceptual understanding: CCR standards call for conceptual understanding of key concepts, such as place value and ratios. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.

Procedural skill and fluency: CCR standards call for speed and accuracy in calculation. Students are given opportunities to practice core functions such as single-digit multiplication so that they have access to more complex concepts and procedures.

Application: CCR standards call for students to use math flexibly for applications in problem-solving contexts. In content areas outside of math, particularly science, students are given the opportunity to use math to make meaning of and access content.

High-level Summary of Major Work in Grades K-8

K–2	Addition and subtraction—concepts, skills, and problem solving; place value
3–5	Multiplication and division of whole numbers and fractions—concepts, skills, and problem solving
6	Ratios and proportional relationships; early expressions and equations
7	Ratios and proportional relationships; arithmetic of rational numbers
8	Linear algebra and linear functions

*For a list of major, additional, and supporting clusters by grade, please refer to 'Focus in Math' at achievethecore.org/focus

INSTRUCTIONAL PRACTICE GUIDE

MATH
SUBJECT

K–8
GRADES

Date

Teacher Name

School

Grade / Class Period / Section

Topic / Lesson / Unit

About The Instructional Practice Guide

Content-specific feedback is critical to teacher professional development. The Instructional Practice Guide (IPG) is a K–12 classroom observation rubric that prioritizes what is observable in and expected of classroom instruction when instructional content is aligned to college- and career-ready (CCR) standards, including the Common Core State Standards (CCSS), in Mathematics (corestandards.org/Math). It purposefully focuses on the limited number of classroom practices tied most closely to content of the lesson.[1]

Designed as a developmental rather than an evaluation tool, the IPG supports planning, reflection, and collaboration, in addition to coaching. The IPG encompasses the three Shifts by detailing how they appear in instruction:[2]

Focus strongly where the standards focus.

Coherence: Think across grades and link to major topics within grades.

Rigor: In major topics, pursue conceptual understanding, procedural skill and fluency, and application with equal intensity.

This rubric is divided into the Core Actions teachers should be taking. Each Core Action consists of indicators which further describe teacher and student behaviors that exemplify CCR-aligned instruction.

Using The Instructional Practice Guide

For each observation, you should make note of what you see and hear. It may be helpful to supplement what you've recorded with further evidence from artifacts such as lesson plans, tasks, or student work. Although many indicators will be observable during the course of a lesson, there may be times when a lesson is appropriately focused on a smaller set of objectives or you observe only a portion of a lesson. In those cases you should expect to not observe some of the indicators and to leave some of the tool blank. Whenever possible, share evidence you collected during the observation in a follow-up discussion.

After discussing the observed lesson, use the Beyond the Lesson Discussion Guide to put the content of the lesson in the context of the broader instructional plan. The questions in the Beyond the Lesson Discussion Guide help delineate what practices are in place, what has already occurred, and what opportunities might exist to incorporate the Shifts into the classroom during another lesson, further in the unit, or over the course of the year.

To further support content-specific planning, practice, and observation, explore the collection of free IPG companion tools, resources, and professional development modules at achievethecore.org/instructional-practice.

1. Refer to Aligning Content and Practice (achievethecore.org/IPG-aligning-content-and-practice) for the research underpinning the Core Actions and indicators of the Instructional Practice Guide and to learn more about how the design of the tool supports content-specific observation and feedback.
2. Refer to Common Core Shifts at a Glance (achievethecore.org/shifts-mathematics) and the K–8 Publishers' Criteria for the Common Core State Standards for Mathematics (achievethecore.org/publisherscriteria-math-k-8) for additional information about the Shifts required by the CCSS.

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CORE ACTIONS AND INDICATORS

For the complete Instructional Practice Guide, go to achievethecore.org/instructional-practice.

MATH SUBJECT   K–8 GRADES

Core Action 1

Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by college- and career-ready standards in mathematics.

A. The enacted lesson focuses on the grade-level cluster(s), grade-level content standard(s), or part(s) thereof.

   Mathematical learning goal: ______________________________

   Standard(s) addressed in this lesson: ______________________________

B. The enacted lesson appropriately relates new content to math content within or across grades.

C. The enacted lesson intentionally targets the aspect(s) of Rigor (conceptual understanding, procedural skill and fluency, application) called for by the standard(s) being addressed.

   Circle the aspect(s) of Rigor targeted in the standard(s) addressed in this lesson: Conceptual understanding / Procedural skill and fluency / Application

   Circle the aspect(s) of Rigor targeted in this lesson: Conceptual understanding / Procedural skill and fluency / Application

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Core Action 2

Employ instructional practices that allow all students to learn the content of the lesson.

A. The teacher makes the mathematics of the lesson explicit through the use of explanations, representations, tasks, and/or examples.

B. The teacher strengthens all students' understanding of the content by strategically sharing students' representations and/or solution methods.

C. The teacher deliberately checks for understanding throughout the lesson to surface misconceptions and opportunities for growth, and adapts the lesson according to student understanding.

D. The teacher facilitates the summary of the mathematics with references to student work and discussion in order to reinforce the purpose of the lesson.

Core Action 3

Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.

A. The teacher provides opportunities for all students to work with and practice grade-level problems and exercises.
   Students work with and practice grade-level problems and exercises.

B. The teacher cultivates reasoning and problem solving by allowing students to productively struggle.
   Students persevere in solving problems in the face of difficulty.

C. The teacher poses questions and problems that prompt students to explain their thinking about the content of the lesson.
   Students share their thinking about the content of the lesson beyond just stating answers.

D. The teacher creates the conditions for student conversations where students are encouraged to talk about each other's thinking.
   Students talk and ask questions about each other's thinking, in order to clarify or improve their own mathematical understanding.

E. The teacher connects and develops students' informal language and mathematical ideas to precise mathematical language and ideas.
   Students use increasingly precise mathematical language and ideas.

If any uncorrected mathematical errors are made during the context of the lesson (instruction, materials, or classroom displays), note them here.

Name: | Date: | Observer Name:

CORE ACTION 1: Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by college- and career-ready standards in mathematics.

INDICATORS / NOTE EVIDENCE OBSERVED OR GATHERED FOR EACH INDICATOR	RATING
A. The enacted lesson focuses on the grade-level cluster(s), grade-level content standard(s), or part(s) thereof. Mathematical learning goal: Standard(s) addressed in this lesson:	Yes- The enacted lesson focuses only on mathematics within the grade-level standards. No- The enacted lesson focuses on mathematics outside the grade-level standards.
B. The enacted lesson appropriately relates new content to math content within or across grades.	Yes- The enacted lesson builds on students' prior skills and understandings. No- The enacted lesson does not connect or has weak connections to students' prior skills and understandings.
C. The enacted lesson intentionally targets the aspect(s) of Rigor (conceptual understanding, procedural skill and fluency, application) called for by the standard(s) being addressed. Circle the aspect(s) of Rigor targeted in the standard(s) addressed in this lesson: Conceptual understanding / Procedural skill and fluency / Application Circle the aspect(s) of Rigor targeted in this lesson: Conceptual understanding / Procedural skill and fluency / Application	Yes- The enacted lesson explicitly targets the aspect(s) of Rigor called for by the standard(s) being addressed. No- The enacted lesson targets aspects of Rigor that are not appropriate for the standard(s) being addressed.

Name:

Date:

Observer Name:

CORE ACTION 2: Employ instructional practices that allow all students to learn the content of the lesson.

INDICATORS[3] / NOTE EVIDENCE OBSERVED OR GATHERED FOR EACH INDICATOR	RATING
A. The teacher makes the mathematics of the lesson explicit through the use of explanations, representations, tasks, and/or examples. ☐ NOT OBSERVED	4- A variety of instructional techniques and examples are used to make the mathematics of the lesson clear. 3- Examples are used to make the mathematics of the lesson clear. 2- Instruction is limited to showing students how to get the answer. 1- Instruction is not focused on the mathematics of the lesson.
B. The teacher strengthens all students' understanding of the content by strategically sharing students' representations and/or solution methods. ☐ NOT OBSERVED	4- Student solution methods are shared, and connections to the mathematics are explicit and purposeful. If applicable, connections between the methods are examined. 3- Student solution methods are shared, and some mathematical connections are made between them. 2- Student solution methods are shared, but few connections are made to strengthen student understanding. 1- Student solution methods are not shared.
C. The teacher deliberately checks for understanding throughout the lesson to surface misconceptions and opportunities for growth, and adapts the lesson according to student understanding. ☐ NOT OBSERVED	4- There are checks for understanding used throughout the lesson to assess progress of all students, and adjustments to instruction are made in response, as needed. 3- There are checks for understanding used throughout the lesson to assess progress of some students; minimal adjustments are made to instruction, even when adjustments are appropriate. 2- There are few checks for understanding, or the progress of only a few students is assessed. Instruction is not adjusted based on students' needs. 1- There are no checks for understanding; therefore, no adjustments are made to instruction.
D. The teacher facilitates the summary of the mathematics with references to student work and discussion in order to reinforce the purpose of the lesson. ☐ NOT OBSERVED	4- The lesson includes a summary with references to student work and discussion that reinforces the mathematics. 3- The lesson includes a summary with a focus on the mathematics. 2- The lesson includes a summary with limited focus on the mathematics. 1- The lesson includes no summary of the mathematics.

3. These actions may be viewed over the course of 2–3 class periods.

Name: Date: Observer Name:

CORE ACTION 3: Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.[4]

INDICATORS[5 6] / NOTE EVIDENCE OBSERVED OR GATHERED FOR EACH INDICATOR / RATING

4- Teacher provides many opportunities, and most students take them.
3- Teacher provides many opportunities, and some students take them; or teacher provides some opportunities and most students take them.
2- Teacher provides some opportunities, and some students take them.
1- Teacher provides few or no opportunities, or few or very few students take the opportunities provided.

A. The **teacher** provides opportunities for all students to work with and practice grade-level problems and exercises. **Students** work with and practice grade-level problems and exercises.	4 3 2 1 ☐ NOT OBSERVED
B. The **teacher** cultivates reasoning and problem solving by allowing students to productively struggle. **Students** persevere in solving problems in the face of difficulty.	4 3 2 1 ☐ NOT OBSERVED
C. The **teacher** poses questions and problems that prompt students to explain their thinking about the content of the lesson. **Students** share their thinking about the content of the lesson beyond just stating answers.	4 3 2 1 ☐ NOT OBSERVED
D. The **teacher** creates the conditions for student conversations where students are encouraged to talk about each other's thinking. **Students** talk and ask questions about each other's thinking, in order to clarify or improve their own mathematical understanding.	4 3 2 1 ☐ NOT OBSERVED
E. The **teacher** connects and develops students' informal language and mathematical ideas to precise mathematical language and ideas. **Students** use increasingly precise mathematical language and ideas.	4 3 2 1 ☐ NOT OBSERVED

If any uncorrected mathematical errors are made during the context of the lesson (instruction, materials, or classroom displays), note them here.

4. There is not a one-to-one correspondence between the indicators for this Core Action and the Standards for Mathematical Practice. These indicators represent the Standards for Mathematical Practice that are most easily observed during instruction.
5. Some portions adapted from 'Looking for Standards in the Mathematics Classroom' 5x8 card published by the Strategic Education Research Partnership (http://math.serpmedia.org/5x8card/).
6. Some or most of the indicators and student behaviors should be observable in every lesson, though not all will be evident in all lessons. For more information on teaching practices, see NCTM's publication Principles to Actions: Ensuring Mathematical Success for All for eight Mathematics Teaching Practices listed under the principle of Teaching and Learning (http://www.nctm.org/principlestoactions).

BEYOND THE LESSON: DISCUSSION GUIDE
MATHEMATICS

INTRODUCTION

The Beyond the Lesson Discussion Guide is designed for the post-observation conversation using the Instructional Practice Guide (achievethecore.org/instructional-practice) or any other observation rubric. The questions put the content of the lesson in the context of the broader instructional plan for the unit or year. The conversation should first reflect on the evidence collected during the observation to consider what worked, what could improve, and what resources are available to support improvement. If any parts of the Lesson Planning Tool (achievethecore.org/lesson-planning-tool) were used in preparing for the lesson, refer to that information during the discussion. After discussing the observed lesson, use the "Beyond the Lesson" questions to help clearly delineate what practices are in place, what has already occurred, and what opportunities might exist in another lesson, further in the unit, or over the course of the year to incorporate the Shifts into the classroom.

1. **Is this unit targeting the Major Work of the Grade? Does the prior unit target Major Work? Does the next unit target Major Work? How much time would you estimate will be spent on the Major Work in this class this year? (K–8)** Focus means significantly narrowing the scope of content in each grade so that students achieve at higher levels and experience more deeply that which remains. For more information on Major Work of the Grade, see achievethecore.org/focus.

2. **Does this unit target the Supporting Work of the Grade? If so, will this unit highlight the connection to the Major Work of the Grade? Explain how. (K–8)** Supporting content enhances Focus and Coherence simultaneously by engaging students in the Major Work of the Grade. For example, materials for K–5 generally treat data displays as an occasion for solving grade-level word problems using the four operations (see 3.MD.3); materials for grade 7 take advantage of opportunities to use probability to support ratios, proportions, and percents.

3. **Summarize how this lesson fits within the unit. Describe how the other lessons and tasks in this unit are intentionally sequenced to help students develop increasingly sophisticated understanding, skills, and practices.** For more information on coherent connections across and within grades, see http://ime.math.arizona.edu/progressions/.

4. **Which of the three aspects of Rigor (conceptual understanding, procedural skill and fluency, and application) are attended to within this unit? If more than one aspect is attended to, when in the unit are they attended to individually, and when are students using them together?** Rigor is defined as pursuing conceptual understanding, procedural skill and fluency, and application with equal intensity. The standards are written using language that informs the reader as to which aspect of Rigor certain standards address. Some clusters or standards specifically require one aspect of Rigor; some require multiple aspects. All aspects of Rigor need not be addressed in every lesson.

5. **How will you meet all students' needs while working on grade/course-level content in this unit? (e.g., How will you provide scaffolding for students below grade/course level so they can reach the grade/course-level expectations? How will you create opportunities for students who are advanced to go deeper into the grade/course-level content?)** For more information, see Adapting the Lesson under Problems & Exercises in the Lesson Planning Tool: achievethecore.org/lesson-planning-tool.

6. **What off-grade/course-level standards have you taught this year and why?** There may be reasons for addressing topics in a strategic way before or after the grade in which the topic is central in the standards. However, any such purposeful discrepancies should enhance the required learning, not unduly interfere with or displace grade/course-level content, and be clearly aimed at helping students meet the standards as written.

7. **In what ways do you provide diagnostic feedback to students? Do students have opportunities to revise their thinking? Does student work include revisions of solutions, explanations, and justifications?**

8. **In what ways have your students made progress towards mastering the grade/course-level content standards? How are you monitoring and tracking their achievement of the standards? What work still needs to be done to ensure all students achieve mastery of each standard by the end of the year?** For more information on the Standards for Mathematical Content, see corestandards.org/Math.

9. **In what ways have you seen your students increase their independence in applying the Standards for Mathematical Practice in learning content this year? Which practice standards do students still need to develop and how can you support them in doing so?** For more information on the Standards for Mathematical Practice, see corestandards.org/Math/Practice.

10. **What tools are appropriate for students to independently access when solving mathematical problems in this unit? Do students frequently choose and use appropriate tools strategically in this class?** For more information on SMP5, see corestandards.org/Math/Practice.

Experience the Content of the Lesson: Discussion Guide *Making Equal Groups* – Grade 3

Ideas that may emerge from the discussion:
Facilitators should choose which points to address based on the needs of the participants and the time allotted for this activity.

- There are 9 unique ways the teacher could group the 36 students equally
    - 1 group of 36 each
    - 2 groups of 18 each
    - 3 groups of 12 each
    - 4 groups of 9 each
    - 6 groups of 6 each
    - 9 groups of 4 each
    - 12 groups of 3 each
    - 18 groups of 2 each
    - 36 groups of 1 each
- The commutative property shows that there are two equations that can be written for each factor pair (e.g., 4 x 9 = 36 and 9 x 4 = 36.) *This idea relates to 3.OA.B.5 Apply properties of operations as strategies to multiply and divide.*
- Students in grade 3 learn to *"interpret products of whole numbers, e.g. interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each."* 3.OA.A.1
- Students will be building fluency with multiplication and division within 100 throughout grade 3 and will know from memory all products of two one-digit numbers by the end of the year. 3.OA.C.7
- Finding all factor pairs for a whole number is aligned to the grade 4 standard 4.OA.B.4 *Find all factor pairs for a whole number in the range 1-100.*

Student Assignment: Participant Handout

Name ______________________

A teacher wants to place her 36 students into groups with an equal number in each group. How many different ways can the teacher group the students?

Show your work.

CCSS WHERE TO FOCUS GRADE 3 MATHEMATICS

MATH 3 F

MATHEMATICS GRADE 3 FOCUS

This document shows where students and teachers should spend the large majority of their time in order to meet the expectations of the Standards.

Not all content in a given grade is emphasized equally in the Standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. More time in these areas is also necessary for students to meet the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the Standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

Students should spend the large majority[1] of their time on the major work of the grade (■). Supporting work (□) and, where appropriate, additional work (○) can engage students in the major work of the grade.[2, 3]

MAJOR, SUPPORTING, AND ADDITIONAL CLUSTERS FOR GRADE 3

Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.

Key: ■ Major Clusters □ Supporting Clusters ○ Additional Clusters

3.OA.A	■	Represent and solve problems involving multiplication and division.
3.OA.B	■	Understand properties of multiplication and the relationship between multiplication and division.
3.OA.C	■	Multiply and divide within 100.
3.OA.D	■	Solve problems involving the four operations, and identify and explain patterns in arithmetic.
3.NBT.A	○	Use place value understanding and properties of operations to perform multi-digit arithmetic.
3.NF.A	■	Develop understanding of fractions as numbers.
3.MD.A	■	Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
3.MD.B	□	Represent and interpret data.
3.MD.C	■	Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
3.MD.D	○	Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
3.G.A	□	Reason with shapes and their attributes.

HIGHLIGHTS OF MAJOR WORK IN GRADES K–8

K–2	Addition and subtraction – concepts, skills, and problem solving; place value
3–5	Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving
6	Ratios and proportional relationships; early expressions and equations
7	Ratios and proportional relationships; arithmetic of rational numbers
8	Linear algebra and linear functions

REQUIRED FLUENCIES FOR GRADE 3

3.OA.C.7	Single-digit products and quotients (Products from memory by end of Grade 3)
3.NBT.A.2	Add/subtract within 1000

1 At least 65% and up to approximately 85% of class time, with Grades K–2 nearer the upper end of that range, should be devoted to the major work of the grade. For more information, see Criterion #1 of the K–8 Publishers' Criteria for the Common Core State Standards for Mathematics www.achievethecore.org/publisherscriteria.
2 Refer also to criterion #3 in the K–8 Publishers' Criteria for the Common Core State Standards for Mathematics www.achievethecore.org/publisherscriteria.
3 Note, the critical areas are a survey of what will be taught at each grade level; the major work is the subset of topics that deserve the large majority of instructional time during a given year to best prepare students for college and careers.

CCSS
WHERE TO FOCUS
GRADE 3
MATHEMATICS

An important subset of the major work in grades K–8 is the progression that leads toward middle school algebra.

K	1	2	3	4	5	6	7	8
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multidigit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations** Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities

* Indicates a cluster that is well thought of as a part of a student's progress to algebra, but that is currently not designated as major by the assessment consortia in their draft materials. Apart from the one asterisked exception, the clusters listed here are a subset of those designated as major in the assessment consortia's draft documents.
** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

The Observation and Feedback Cycle: Best Practices for Low Inference Notes

Observe

The school leader visits the classroom and takes low-inference notes on teacher and student actions.

Best Practices for Observation
1. **Eliminate effects of bias.** Enter the classroom without judgment and work from evidence.
2. **Take low-inference notes.** Write down only what teacher and students say and do.
3. **Look for learning.** Seek evidence of what students know and are able to do.
4. **Remain, review, reflect.** Pause to organize your evidence before rating.

Collecting low inference evidence during an observation

Capturing high-quality notes during the observation is the first step in ensuring that ratings are accurate and feedback aligns to teachers' needed areas of improvement. **Low-inference note-taking is a skill**, not knowledge. Knowing how to do a push-up doesn't mean you can do 25 of them in 60 seconds; it comes with practice. When taking low-inference notes, the school leader describes what is taking place without drawing conclusions or making judgments about what he or she observes. When taking notes on instruction, ask:

- What do you see and hear the teacher and students saying and doing?
- What evidence can you gather of student learning?
- What will students know and be able to do at the end of the lesson?

Common mistakes/pitfalls to avoid

- Distinguish between low-inference statements and opinions. For instance, you can identify key words that give away subjectivity: e.g., *"I think,"* or *"I feel."* Be cognizant of keeping evidence separate from opinions, using this framework:

Evidence	Opinion
• Is observable • Is not influenced by the observer's perspective • Is free of evaluative words • Does not draw conclusions	• Makes inferences • Depends on observer's perspective • Includes evaluative words • Draws conclusions

- Replace vague quantifiers by capturing more specific evidence: e.g., *"a lot of students raised their hands"* vs. *"17 of 20 students raised their hands."*
- Swap Edu-Speak for Evidence. For example, rather than saying, *"You differentiated by scaffolding questions during the mini-lesson,"* identify the actual questions that the teacher asked, such as *"What is the name of this shape? How is it different from a square or rectangle? Where in real life have you seen this shape?"*

For electronic version, visit: https://www.weteachnyc.org/media2016/filer_public/2a d3/2ad3839feeae-42ba-8249-614b33136717/ best_practices_for_low_inference_notes.pdf

Tips for low inference note taking

Where to find the data for student outcomes during an observation:

- Sit with a table/group of students. Write down the questions asked and answers given by the students in that group.
- Copy down what each student has written on their paper VERBATIM into your observation notes (e.g., answer to #2 on handout, response to quick-write prompt). The observer can obtain a handout from the teacher, if available, and record the answers directly onto it.
- Write down the time and circulate in the room. Record the item that all students are working on in that moment. Then, go around a second time.
- Select a problem, determine the correct answer, and tally the number of students who have the correct response written on their papers.
- If recording observation notes using an iPad, use the iPad to take pictures of actual student work during the classroom observation.
- Move around the classroom and identify students performing at high, medium, low levels and strategically capture their work
- Monitor observation notes to ensure that the “student side” is not neglected.
- Ask students to tell you what they are learning/doing, why they are learning, and if they have learned anything new today.
- Collect the lesson plan and/or copies of student work prior to leaving the classroom.

How do I capture as much evidence as possible?

- Set up a coding system (T= teacher, S= student, HU= hands up)
- Time transitions, each section of the lesson, work time, etc.
- Copy objective or aim, or make a note if it is not posted
- Draw circles to represent groups of students or teacher interaction with students

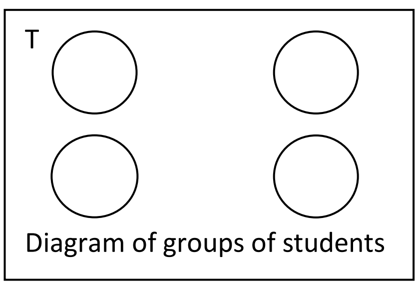
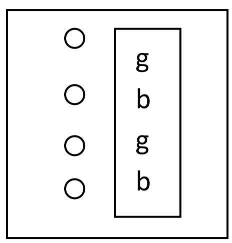


Diagram of groups of students



- If you notice a trend, create a tally on the side, so you can capture other evidence that may be occurring while also documenting the trend. For example, Jane is the only one responding to the teacher’s questions. You may capture several instances verbatim, but you can also capture how many times it occurs if you can’t capture everything Jane said.

  Use tallies or shorthand in the diagram or a chart:

Jane is called on	~~IIII~~ I
Times teacher provides feedback to front table	III

- Quality over quantity: collect a full interaction.
  - *When teacher did __, student __. When student said __, teacher said__.*

Low-Inference Note-Taking Samples: Strong versus Weak

Strong example of low-inference notes:

Time	Teacher Actions	Student Actions
1:00	Teacher says to walking students, "You need to be on the rug in 3-2-1."	Twenty-four students on the carpet facing the front of the room. 3 students walking around the classroom. As teacher said "one" students joined classmates.
1:01	Teacher asked "How many days are there in the week?" Teacher repeated question and then said, "Anyone?" Teacher asked kids to stand and lead them in "The Days of the Week" song.	5-6 kids spoke to each other when teacher spoke. She called on Terrence who said "7." 16 of the 27 kids stood up for the song.
1:02	Teacher asked "What day comes after Saturday?"	Steven shouted out, "Monday!" Most students laughed – 2 boys physically rolled around and knocked over 2 girls. Steven walked away from the group, and sat in the opposite corner of the classroom.
1:03	Teacher said, "OK boys and girls if you hear my voice clap once, If you hear my voice clap twice."	After two claps, all but 2 boys were quiet and looking at her.

Weak example of low-inference notes:

Time	Teacher Actions	Student Actions
1:00		Students on carpet during mini-lesson. Lots of students walking around the classroom while the teacher tried to get their attention.
1:01	Teacher asked questions about the calendar.	Many students were not listening while the teacher reviewed the days of the week.
1:02		Steven called out over and over again when you asked the question about the days of the week.
1:03		Steven walked away from the group and the class fell apart.
1:04	Mini-lesson is not successful. Little student learning accomplished as teacher has no classroom management skills.	
1:05	Poor classroom management continues through sloppy transitions from carpet to desks.	Several students are talking to one another.
1:06	The teacher seemed to be okay with this.	A few students go to the round table. Some start reading and some don't.

Instructional Practice Guide – Making Equal Groups (Grade 3): Model Response

Indicator	Evidence
Core Action 1: Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by college- and career-ready standards in mathematics.	
A. The enacted lesson focuses on the grade-level cluster(s), grade-level content standard(s), or part(s) thereof.	Mathematical learning goal: • The learning goals explicitly explained to students (~2:20) are "to persevere through math," "using drawings, possibly equations, or even math tools" to solve problems with equal groups, and "to possibly use some math vocabulary as we're explaining our thinking." The mathematical goal referred to during the lesson is "exploring equal groups" (~28:28). Standard(s) addressed in this lesson: • The work of the lesson—both the mathematics that students engage with and the focus of the debrief—aligns closest to the cluster 3.OA.A: Represent and solve problems involving multiplication and division. • The problem, as written, aligns to a grade 4 standard (4.OA.B.4) because students are asked to find all factor pairs of a whole number. This could have taken the lesson off grade level, but the teacher used it in a way that reinforced grade-level expectations around representations of products and factors.

Indicator	Evidence
B. The enacted lesson appropriately relates new content to math content within or across grades.	• The teacher activates students' prior skills and knowledge around multiplication and division. • The teacher launches the lesson by asking students to talk about their work the previous day with even and odd factors and products and the teacher uses this as an opportunity to reinforce vocabulary (~0:10). It is unclear whether students were expected to use the concept of even and odd in the work of this lesson. • The teacher also says, "This isn't anything new; we've worked with equal groups for a while...so we're coming back to...rediscover and connect some of those learnings and discoveries we had earlier in the school year to now." (~2:15) From the student work and responses, it is clear that students were applying prior work with the concepts of multiplication and division as well as different visual models to use for equal groupings and applied that understanding to this lesson.
C. The enacted lesson intentionally targets the aspect(s) of Rigor (conceptual understanding, procedural skill and fluency, application) called for by the standard(s) being addressed.	Aspect(s) of Rigor targeted in the standard(s) addressed in this lesson: • Standard: The standards in the 3.OA.A cluster are primarily targeting conceptual understanding with some calling for application. Aspect(s) of Rigor targeted in this lesson: • Lesson: primarily conceptual understanding and secondarily application. - The work of the lesson is based on the application problem: "A teacher wants to place her students in equal groups with an equal number in each group. How many different ways can you make the groups?" The context of the problem is aligned to grade 4; it was not attended to in the discussion of solution strategies. Therefore, the lesson primarily targets conceptual understanding because students were interpreting products and quotients and translating between different representations.

Indicator	Evidence
Core Action 2: Employ instructional practices that allow all students to learn the content of the lesson.	
A. The teacher makes the mathematics of the lesson explicit through the use of explanations, representations, tasks, and/or examples.	• The specific mathematical purpose of the lesson is not clear from the teacher's examples and representations. At some times, the mathematical idea seems to be about representing the same arrangement in different ways (~17:03, ~19:44) and other times it seems to be about figuring out multiple ways to make equal groups of 36 objects (in this case, students). (~3:18, ~28:28)
B. The teacher strengthens all students' understanding of the content by strategically sharing students' representations and/or solution methods.	• The teacher takes notes of the different solutions and representations students are using during individual work time and seems to strategically choose which students will share based on those observations. (~7:14) • It appears that the sequence of the shares was strategic (the point of the share was to show there were different ways to represent with concrete or visual models) but there isn't evidence that this helped all students deepen their understanding of the concepts in 3.OA.A. When the teacher had students share their work (~13:14), she began with a student who used tally marks to show 6 groups of 6. She focused that part of the discussion on the choice to use tally marks. She then had a student show how to use cubes to make 9 groups of 4 and used a representation to show 9 circles with 4 dots in each. Then, she had a student who made arrays come up and share. It's possible that the point of the share was to show there were different ways to represent with concrete or visual models, but there wasn't a clear mathematical point about why it mattered which model to use, whether there was a benefit to any of them, and how the solution connected to the context of the problem.
C. The teacher deliberately checks for understanding throughout the lesson to surface misconceptions and opportunities for growth, and adapts the lesson according to student understanding.	• The teacher deliberately checks for understanding with individual students throughout the individual work time (~4:00–11:15). It is difficult to determine from the observation alone whether any adjustments were made based on these checks. • The teacher sees and addresses misconceptions students have about the problem (~4:17) and support them in thinking about the concept of equal groups (~9:45).

Indicator	Evidence
D. The teacher facilitates the summary of the mathematics with references to student work and discussion in order to reinforce the purpose of the lesson.	• The lesson includes a short summary with references to student work and discussion, but there is a limited focus on the grade-level mathematics. The summary is more focused on vocabulary and whether students engaged in specific Standards for Mathematical Practice with questions like, "Did we come up with different ways to solve the problem? Did you persevere? Did you use drawings and equations?" (~28:25)
Core Action 3: Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.	
A. The teacher provides opportunities for all students to work with and practice grade-level problems and exercises. Students work with and practice grade-level problems and exercises.	• The lesson focuses on a single task and the mathematical work is deep enough to engage all students in grade-level work throughout the lesson. All students are working with the grade 3 work of solving word problems in situations involving equal groups. Students are using manipulatives as a way to represent the concept of multiplication as equal groups and the language "equal groups" is repeated throughout the lesson. There are also opportunities to connect the equal groups of objects representations to written equations.
B. The teacher cultivates reasoning and problem solving by allowing students to productively struggle. Students persevere in solving problems in the face of difficulty.	• Students spend a considerable amount of time working on the problem, which may be evidence of perseverance. • The teacher asks students questions such as, “What’s going to be your next thought with an equal group.” (~5:47) and “How else could you use these to help you find another equal group?” (~9:58). • In conferencing with individual students, the teacher encourages students who are stuck to try out different-sized equal groups and see what works. The teacher conferenced with a student and asked, "So you're just kind of trying it, seeing if it works and if not, moving on to the next one?" (~7:55) • The line of questioning about seeing relationships between 6x6 and 3x12 offered a challenge for students who were ready for it. (~21:15)
C. The teacher poses questions and problems that prompt students to explain their thinking about the content of the lesson.	• The teacher poses a problem that provides an opportunity to understand where students are in their understanding and fluency with multiplication and division. The task, along with the extended work time and student choice of tools, allows the

Indicator	Evidence
Students share their thinking about the content of the lesson beyond just stating answers.	teacher to understand where students are in their developing thinking of multiplication and division. • There are many occasions where the teacher prompts students to share their thinking about their work (~8:19, ~14:03, ~17:30). In some of these cases, the students respond with nod or shrug and the teacher either asks another question or answers it for the students. • The teacher asks many "What…" questions during the student work time and sharing that prompt students to explain their thinking, but no "Why…" questions that prompt students to explain the content of multiplication and division as related to the problem. • One student explains how splitting a 3 x 12 array in half leads to 6 groups of 6 (~24:56). Another student elaborates by connecting the lesson to previous work with even numbers. (~27:24)
D. The teacher creates the conditions for student conversations where students are encouraged to talk about each other's thinking. Students talk and ask questions about each other's thinking, in order to clarify or improve their own mathematical understanding.	• The teacher is working to create a culture where students ask questions about each other's thinking by reminding students to ask clarifying questions of their partners during paired discussions. (~12:20) • During whole group discussion, most questions are asked by the teacher. (~18:15) • When a student is presenting at the end, other students are asking questions and really trying to make sense of his work: "How does that make 36?"; "I looked at it and saw you could split it into 3's too." (~24:27)
E. The teacher connects and develops students' informal language and mathematical ideas to precise mathematical language and ideas. Students use increasingly precise mathematical language and ideas.	• At the beginning of the lesson, the teacher and students use precise language for even and odd numbers as well as the terms "products" and "factors" (~00:24). A vocabulary list is created for the class to note mathematical terms used throughout the lesson. As students describe their solutions and the teachers asks questions, the terms "product" and "factors" are not used; however, they are reinforced again and connected to the targeted task at the end of the lesson. (~28:58) • There are occasions where the teacher does not attend to students' informal language. Teacher does not clarify when student shares, "I just did it on a bunch of numbers and I used

Indicator	Evidence
	the cubes to see if it worked." (~15:05) and “he did 6 times 6 and then went up and the up part equals 36.” (~14:14) • When a student explains how splitting a 3 x 12 array in half leads to 6 groups of 6 (~24:56) and the class is grappling with the mathematical ideas the teacher ends the discussion with “So we have a lot to think about now, don’t we?” (~28:20) rather than connecting some of the informal ideas shared to more precise mathematical language or ideas.

If any uncorrected mathematical errors are made during the context of the lesson (instruction, materials, or classroom displays), note them here:

- No mathematical errors were observed.

Teacher-created Lesson Plan

CCSS Math Standard:

3.OA.A.3 Represent and solve problems involving multiplication and division: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Aspects of rigor called for by the standard being addressed in this lesson:

- Application – primary
- Conceptual Understanding – secondary

Lesson Plan

Materials:

- Student Math Journals
- Math tools: Base Ten blocks, grid paper, geoboards, etc.
- Chart paper – problem to solve
- Individual stickers with problem for Student Math Journals

Student Objectives:

- To identify as many equal groups as possible to a number using a method of choice to assist a student in solving the problem. (e.g., arrays, counters, pictures, etc.)
- To find multiple answers to a problem.
- To engage in a partner, small group, and whole class discussion explaining and justifying students' math thinking.

Problem to Solve:

A teacher wants to place her 36 students into groups with an equal number in each group. How many different ways can the teacher group the students?

Lesson:

1. Begin the lesson with discussing the following with students:
   a. Our learning goal is to: (written on the board)
      i. To persevere in solving math problems that involve equal groups.
      ii. To use drawings, equations, and/or math tools to solve math problems that involve finding equal groups.
      iii. To use mathematical vocabulary such as factor, product, and array in justifying answers in math solutions.
   b. You will be asked to record your thinking and reasoning in your Math Journal.
2. Pose the problem to the class:
   A teacher wants to place her 36 students into groups with an equal number in each group. How many different ways can the teacher group the students?
3. As students begin solving the problem, give each student a label with the problem on it for the label to be placed in their Math Journal.
4. While students work through the problem, walk around the room to observe student thinking and problem solving.
5. Once students have begun problem solving, ask individual students to explain his/her thinking to you.

   a. Look for – math tool (student choice) for solving, persevering to find all the different ways to equal groups of 36, and/or student thinking and reasoning.
      i. Groups of 36: 1 x 36 = 36, 36 x 1 = 36, 2 x 18 = 36, 18 x 2 = 36, 3 x 12 = 36, 12 x 3 = 36, 4 x 9 = 36, 9 x 4 = 36, and 6 x 6 = 36.

6. Discussion of Solutions and Strategies:
   a. Invite different students to present their solutions and/or strategies to the problem.
   b. As a student presents, open class to a Whole Class Discussion to ask questions for clarification, understanding, and/or extensions to a solution/strategy.
   c. Present patterns to solutions and discuss with the class.
      i. Look for levels in problem representation and solution (as stated in the Progression Documents):
         1. Level 1: Making and counting all quantities in multiplication and division with objects or a diagram. Student uses either the objects or diagram to explain thinking. This would include arrays and counting of objects.
         2. Level 2: Repeated counting on by a given number, such as 4, 8, 12, 16, 20, 24, 28, 32, 36. Student counts by fours and tracks how many fours are used. In this case it would be 9 fours are used to make 36. Equations may also count in this category such as 4 x 9 = 36. Using a number line may also be an option.
         3. Level 3: Use of the associative or distributive property to compose and decompose numbers in finding a solution. For example: 3 x 12 = 3 x (3 x 4) = (3 x 3) x 4 = 36. Students may decompose a product they do not know to one that they do know and then build from there. Students may also know a product 1 or 2 ahead or behind a given product and say: I know 3 x 11 = 33, so 3 x 12 is 33 + 3.

7. <u>Review the learning goal:</u>
   a. Ask: What was the first step you took in solving this problem of finding equal groups? Have students partner share and then call on one/two students to share.
   b. Ask: Describe one or two things you learned from another person's justification. Again, have students share with a partner and ask a few students to share with whole class.
   c. Ask: Was a solution or strategy presented today that you had not thought about using that you might use the next time you will find equal groups?
      i. Have those students share their thinking.

8. <u>Closing:</u>
   a. As a class, we will continue to find equal groups in real world problems and I encourage you to build upon your knowledge from your work today to help you with future problems.

<u>Important Facts:</u>

- Student grouping has been strategic. There is a high group that continues to possess productive challenges amongst themselves that has been intriguing to observe. There are a few struggling students in persevering that have been grouped together so they have to rely on him/herself to find a solution. The goal is to help these students build confidence as well as persevering. Some students have been grouped with a partner that they feel most comfortable with for discussion/explanation.

Lesson Plan Analysis

Lesson: ___________________________________

Use this document to record information/evidence from the sample lesson plan. Evidence should consider the Core Actions. Evidence recorded will be integrated into the Feedback Summary worksheet.

Core Action 1: Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by the college- and career-ready standards in mathematics.
Discussion Questions
• **Which standard(s) and/or cluster(s) are targeted in this lesson? Does the lesson address a part of the standard(s) or all aspects of the standard(s)? Are they grade-level standards? Are they part of the Major Work of the grade?** • **If the standard(s) targeted are Supporting Work of the grade, how will connections be made to engage students in the Major Work of the grade?** • **What is the mathematical learning goal for students in this lesson?** • **Which aspect(s) of Rigor (conceptual understanding, procedural skill and fluency, and application) do the targeted standards require? What features of the lesson support the aspect(s) of Rigor present in the targeted standards?** • **How does the teacher plan to make explicit connections to build on students' prior skills and understandings? What will the teacher say to students or show students to make this connection clear?**

Core Action 2:
Employ instructional practices that allow all students to learn the content of the lesson.

Discussion Questions

- **How does the teacher plan to use explanations, representations, tasks, and/or examples that will make the mathematics of this lesson clear to students?**
- **What will students produce? Are they expected to produce only answers?**
- **What ideas/concepts will be the focus of discussions?**
- **How will students share/present their mathematical work to support all students' understanding of the topic?**
- **When in the lesson does the teacher plan to check for understanding?**
- **How does the teacher plan to summarize the mathematics of the lesson? Will the summary include student work and discussion to reinforce the mathematical learning goal of the lesson?**

Core Action 3: Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.

Discussion Questions

- **What mathematical language will be used in this lesson? How will the teacher support students' use of increasingly precise language, including for English language learners if applicable?**
- **Are mathematical models, mathematical representations, mathematical arguments, and mathematical counter-arguments expected from students, as required by the standards? What problem(s) and question(s) will allow students to share their thinking and/or justify their conclusions?**
- **When will students be doing grade-level problems and exercises? Will all students have this opportunity?**

Lesson Plan Analysis: Model Response

Lesson: Grade 3 - Making Equal Groups

Use this document to record information/evidence from the sample lesson plan. Evidence should consider the Core Actions. Evidence recorded will be integrated into the Feedback Summary worksheet.

Core Action 1: Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by the college- and career-ready standards in mathematics.

Discussion Questions

- **Which standard(s) and/or cluster(s) are targeted in this lesson? Does the lesson address a part of the standard(s) or all aspects of the standard(s)? Are they grade-level standards? Are they part of the Major Work of the grade?**
- **If the standard(s) targeted are Supporting Work of the grade, how will connections be made to engage students in the Major Work of the grade?**
- **What is the mathematical learning goal for students in this lesson?**
- **Which aspect(s) of Rigor (conceptual understanding, procedural skill and fluency, and application) do the targeted standards require? What features of the lesson support the aspect(s) of Rigor present in the targeted standards?**
- **How does the teacher plan to make explicit connections to build on students' prior skills and understandings? What will the teacher say to students or show students to make this connection clear?**

The lesson plan names a targeted standard, student objectives, and learning goals. While the plan identifies 3.OA.A.3 (Major Work of grade 3) as the targeted standard, two of the three student objectives ("To identify as many equal groups possible to a number using a method of choice to assist a student in solving the problem. (e.g., arrays, counters, pictures, etc.)" and "To find multiple answers to a problem") are more closely aligned to the factoring work required in grade 4 (4.OA.B.4). The Learning Goals are focused more strongly on the Standards for Mathematical Practice than the grade 3 content standards (e.g., "To use mathematical vocabulary such as factors, product, and array in justifying answers in math solutions").

3.OA.A.3 primarily targets application, with its reference to real-world problems and secondarily targets conceptual understanding as part of the 3.OA.A cluster. The lesson begins with a real-world application problem. The discussion, as laid out in the plan, tends to focus on the conceptual ideas about representing multiplication and understanding the connections between representations.

In part 6c of the lesson plan, the teacher references the progression of multiplication representations and strategies, as defined in the OA Progression document. This information will be helpful for the teacher as she observes and conferences with students and leads a whole group discussion of the problem. In this way, the plan references how students' strategies for solving multiplication problems should get more sophisticated as the year goes on, to ensure that students meet the fluency requirement of 3.OA.C.7 by the end of grade 3.

Core Action 2:
Employ instructional practices that allow all students to learn the content of the lesson.

Discussion Questions

- **How does the teacher plan to use explanations, representations, tasks, and/or examples that will make the mathematics of this lesson clear to students?**
- **What will students produce? Are they expected to produce only answers?**
- **What ideas/concepts will be the focus of discussions?**
- **How will students share/present their mathematical work to support all students' understanding of the topic?**
- **When in the lesson does the teacher plan to check for understanding?**
- **How does the teacher plan to summarize the mathematics of the lesson? Will the summary include student work and discussion to reinforce the mathematical learning goal of the lesson?**

The teacher plan shows a task that allows for multiple entry points for students who have already done work with multiplication and division. The problem allows students to identify and represent different multiplication expressions with a product of 36. Students are encouraged to use a variety of models and representations to share their work. Although the task explicitly asks students to make a claim about how many possible arrangements there are (and section 5a.i lists all the combinations), that question doesn't seem to come up during the remainder of the lesson plan. The plan notes that the teacher will observe students working and then ask them questions so they can explain their thinking. This could serve as a check for understanding and a way to prepare for the whole class discussion.

After individual work time, the plan states that students will present solutions and strategies to the problem and that there will be a whole class discussion to allow for questions. It also states that the teacher will "[P]resent patterns to solutions and discuss with the class." This is followed by a list of the levels of solving that students will move through to become fluent with multiplication. The inclusion of these levels (with ties to the specific problem) implies that the discussion will focus on connecting representations and moving students toward more efficient strategies.

The plan includes a summary of the lesson where students reference the strategies they saw in their classmates' work (e.g., "Describe one or two things you learned from another person's justification"). The next question is a way to encourage students to use more sophisticated strategies as they continue their work with multiplication and division.

Core Action 3:
Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.

Discussion Questions

- **What mathematical language will be used in this lesson? How will the teacher support students' use of increasingly precise language, including for English language learners if applicable?**
- **Are mathematical models, mathematical representations, mathematical arguments, and mathematical counter-arguments expected from students, as required by the standards? What problem(s) and question(s) will allow students to share their thinking and/or justify their conclusions?**
- **When will students be doing grade-level problems and exercises? Will all students have this opportunity?**

One of the student-facing learning goals mentions using vocabulary, specifically "factor, product, and array"; these words are not referenced in the plan beyond the learning goal. There is time planned for structured conversation in part 7 that may be intended to support English language learners.

Students will be expected to use mathematical models and representations as noted by the learning goal notes, "To use drawings, equations, and/or math tools to solve math problems that involve finding equal groups."

Student A

Name [redacted]

A teacher wants to place her 36 students into groups with an equal number in each group. How many different ways can the teacher group the students?

Show your work.

$18 + 18 = 36$

$12 + 12 + 12 = 36$

$6 \times 6 = 36$

Student B

Name [redacted]

A teacher wants to place her 36 students into groups with an equal number in each group. How many different ways can the teacher group the students?

Show your work.

$\boxed{6} \times 6 = 36$ student

6

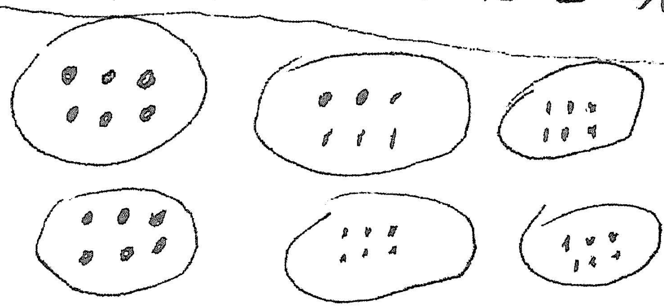
+6 +6 +6 +6 +6

12 18 24 30 36

$6 \times 6 = \boxed{36}$

$6+6+6+6+6+6 = \boxed{36}$

$12 + 12 + 12 = 36$



6

$36 \div 6 = \boxed{6}$

36

−6 −6 −6 −6 −6 −6

6 12 18 24 30

Student C

Name [redacted]

A teacher wants to place her 36 students into groups with an equal number in each group. How many different ways can the teacher group the students?

Show your work.

36×1=36 18×2=36 14 16 18 20 22 24 26 28 30 32 34 36

2 4 6 8 10 12 12 × 16 × 20 × 24 × 28 × 32 × 36

4 × 8 × 12 × 16 × 18 × 24 × × 30 × × 36

9×4=36 × 8 × × 16 × × × 24 × × × 32 ×

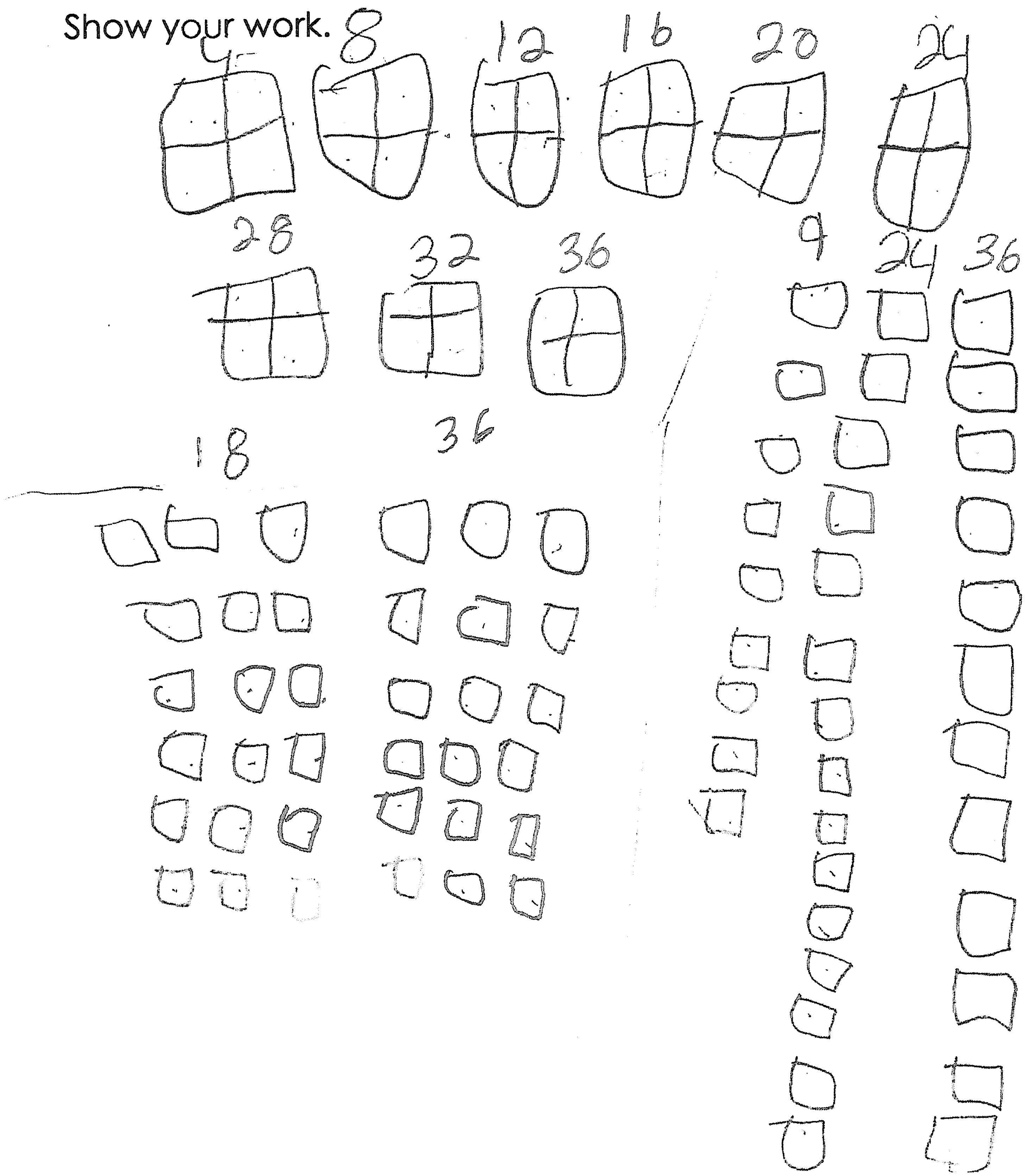
6×6=36

Student D

Name [redacted]

A teacher wants to place her 36 students into groups with an equal number in each group. How many different ways can the teacher group the students?

Show your work.



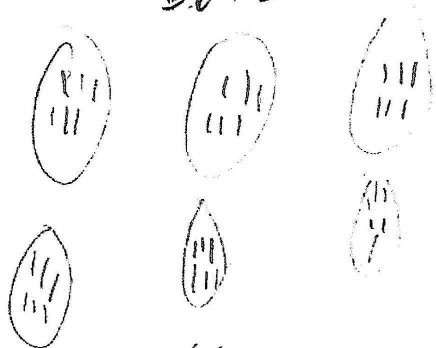
Student E

Name [redacted]

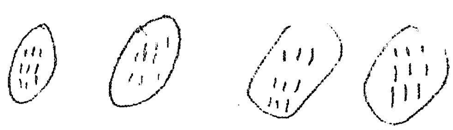
A teacher wants to place her 36 students into groups with an equal number in each group. How many different ways can the teacher group the students?

Show your work.

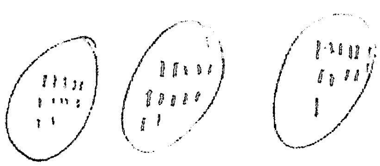
$36 \div 6 = 6$



9X4=36



12X3=36



6X6=36

1X36=36

Student F

Name [redacted]

A teacher wants to place her 36 students into groups with an equal number in each group. How many different ways can the teacher group the students?

Show your work.

6 12 18 1.

24 30 36

5 10 15 20 25 30 35 $+1=36$ 2.

3.

6	# in each	T
6	6	36

$36 \div 6 = 6$

2 6 10 14 $14 \times 2 = 36$ 4.

4 8 12 16 18 20 22 24 26

Student G

Name [redacted]

A teacher wants to place her 36 students into groups with an equal number in each group. How many different ways can the teacher group the students?

Show your work.

$9 \times 4 = 36$

9 groups of 4 = 36 students

8

1234 + 1234 + 1234

8 + 9

1234 + 1234 + 1234

8

1234 + 1234 + 4 1234

Groups 9

$8+8=16$

$8+8=16$

$16+16=32+4=36$

$6+6=12$

$12+10=22+10=32$

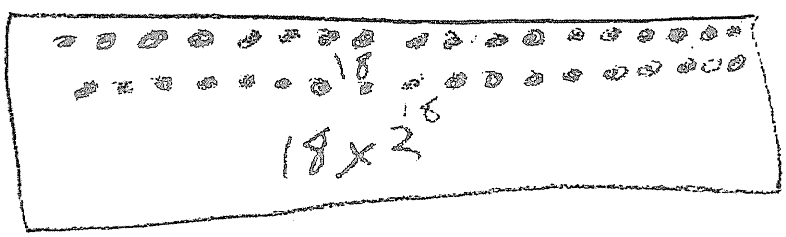
Student H

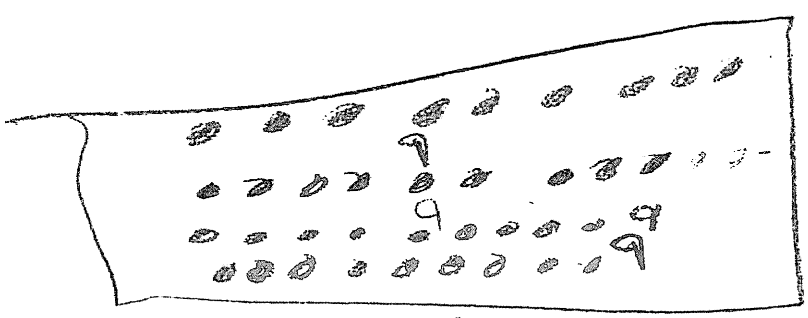
Name [redacted]

A teacher wants to place her 36 students into groups with an equal number in each group. How many different ways can the teacher group the students?

Show your work.

12x3





9x4=36

4x9

Student Work Analysis

Lesson: ____

Use this document to record information/evidence from the sample student work. Evidence should consider the Core Actions. Evidence recorded will be integrated into the Feedback Summary worksheet. ***Before analyzing student work, be sure to have first completed the student assignment.***

General notes and observations about the task:

1. **Which standard(s) and/or cluster(s) are targeted in this assignment? Are they grade-level standards?**

2. **What is the mathematical purpose of the assignment?**

3. **What aspect(s) of Rigor (conceptual understanding, procedural skill and fluency, and application) does the assignment address? Explain.**

Analyzing individual student samples (worksheet on next page):

1. **What does the student's work demonstrate about his/her understanding of the expectations of the assignment?**
2. **What does the student's work demonstrate about his/her proficiency with the requirements of the targeted standard?**

(See worksheet)

Student Work Analysis Worksheet

Student Work Sample	What does the student's work demonstrate about his/her understanding of the expectations of the assignment?	What does the student's work demonstrate about his/her proficiency with the requirements of the targeted standard?
Student <u>A</u>		
Student <u>B</u>		
Student <u>C</u>		
Student <u>D</u>		

Note: For a collection of more than four samples of student work, print this page multiple times.

After looking at student work:

1. **How did the directions and/or prompts for the assignment allow students to demonstrate the requirements of the targeted standard(s)?**

2. **How did the mathematical content of the assignment allow students to demonstrate the requirements of the targeted standard(s)?**

3. **What patterns do you notice in the student work?**
    - **What did students do consistently well?**
    - **Were there any common errors?**

Student Work Analysis: Model Response

Lesson: Grade 3 - Making Equal Groups

Use this document to record information/evidence from the sample student work. Evidence should consider the Core Actions. Evidence recorded will be integrated into the Feedback Summary worksheet. ***Before analyzing student work, be sure to have first completed the student assignment.***

General notes and observations about the task:

1. **Which standard(s) and/or cluster(s) are targeted in this assignment? Are they grade-level standards?**
   As written, the task asks students to find all the factor pairs for 36, which aligns to the first part of 4.OA.B.4 ("Find all factor pairs for a whole number in the range 1–100") which is above grade level.

2. **What is the mathematical purpose of the assignment?**
   The task requires students to determine how many different ways 36 students can be grouped into equal groups.

3. **What aspect(s) of Rigor (conceptual understanding, procedural skill and fluency, and application) does the assignment address? Explain.**
   The task is an application problem that requires students to apply conceptual understanding of multiplication and division to a real-world situation.

   Note: Although the task as written aligns to 4.OA.B.4, since it was used in a grade 3 class, the analysis of student work will consider cluster 3.OA.A ("Represent and solve problems involving multiplication and division") as the targeted standard.

Analyzing individual student samples (worksheet on next page):

1. **What does the student's work demonstrate about his/her understanding of the expectations of the assignment?**
2. **What does the student's work demonstrate about his/her proficiency with the requirements of the targeted standard?**

(See worksheet)

Student Work Analysis Worksheet

Student Work Sample	What does the student's work demonstrate about his/her understanding of the expectations of the assignment?	What does the student's work demonstrate about his/her proficiency with the requirements of the targeted standard?
Student A	Student A appears to understand the problem posed (find different ways to arrange 36 students equally).	Student A uses equations to represent different ways to compose 36. Student uses repeated addition to show how many students would be in each group if there were 2 groups or 3 groups; it is not clear how the student determined how many students would be in each group. It is possible that the student did mental computation or just knew the double fact of 6x6.
Student B	Student B seems to misinterpret the directions and uses multiple models and strategies to show that 36 can be divided into 6 equal groups of 6, rather than finding different sized equal groups.	Student B uses multiple strategies and models to represent 6 equal groups of 6 students. He/she uses skip counting, repeated addition and repeated subtraction (modeled on an open number line), and an equal groups representation that includes arrays. For the most part, the student does not relate the mathematics to the context of the problem, although he/she does label 36 students for the first equation.
Student C	Student C uses a systematic way of checking which numbers are factors of 36, which implies that he/she understands what the task is asking. The student uses a table that could help identify patterns in the factors of 36. He/she writes multiplication equations to show combinations of 36.	Student C investigated if 2, 4, 6, and 8 were factors of 36 and determined 2, 4, and 6 were. Based on the equations he/she wrote, the student has identified most of the factors of 36 (36 x 1, 18 x 2, 9 x 4, 6 x 6). It is not clear from the student work whether he/she understands the context of the problem and could articulate that most of the expressions could represent two different scenarios in the context of the problem (e.g., 18 x 2 could mean that there are 2 groups with 18 students in each group or 18 groups with 2 students in each group.) It would be interesting to ask this student why he/she did not investigate whether 3 is a factor of 36.
Student D	Student D appears to understand the problem posed.	The note "Tiles" in the upper right corner seems to indicate that Student D used concrete models and then created a visual to represent the groups. In creating the visual model, he/she labeled the cumulative total of tiles, rather than the amount in each group. On the right side, the student's picture and count do not represent equal groups. The first group has 9 tiles and is labeled correctly. The next two groups have 12 tiles each and the student's cumulative total represents a correct count if he/she were counting by 12.

Student Work Sample	What does the student's work demonstrate about his/her understanding of the expectations of the assignment?	What does the student's work demonstrate about his/her proficiency with the requirements of the targeted standard?
Student <u>E</u>	Student E appears to understand the problem posed.	Student E is able to show 4 different arrangements of 36 (6 groups of 6, 4 groups of 9, 3 groups of 12, and one group of 36). Although the equations aren't always consistently representing the grouping (e.g., 9 x 4 shows 4 groups of 9 but 1 x 36 shows 1 group of 36), the student did represent the visual model with both multiplication and division equations. It would be interesting to ask the student how he/she thought about 1 group of 36 in the context of the problem.
Student <u>F</u>	Student F seems to misinterpret the directions and uses multiple models to show that 36 can be divided into 6 equal groups of 6.	Student F represents 6 equal groups of 6 in several ways: using visual models of equal groups, using a t-chart, and with the division equation 36 ÷ 6 = 6. The student seems to be using skip counting to determine whether 36 can be split equally into groups of 2 and 5. The representation marked with "2" shows 7 groups of 5 with one left over. It is unclear whether the student was testing different-sized groups or if he/she thinks this is a valid answer to the question. The last representation shows 13 x 2 = 26 which is then erased. The student then tried adding one more (14 x 2) and incorrectly concluded 14 x 2 = 36. Although the student seemed to correctly identify 2 as a factor of 36, he/she was not able to use the fact 13 x 2 = 26 to come up with a correct answer to __ x 2 = 36. The student does not relate the mathematics to the context of the problem.
Student <u>G</u>	Student G finds one way to divide the students into equal groups. It is unclear whether he/she knew to try to find more than one way to group the students.	Student G appears to have started by identifying that the multiplication fact (9 x 4) equals 36. It appears as though the first three lines of equations are a way to double check that the 9 groups of 4 are equal to 36. The student starts by adding 2 8s and then another 2 8s to get 32. Then he/she adds the 4 from the diagram to get 36. (Although the equal sign is used incorrectly to say that all three parts of the equation are equal.) The last two lines are using a partial sum strategy to add 16 + 16. The strategy the student uses is based on the distributive property and the understanding that

Student Work Sample	What does the student's work demonstrate about his/her understanding of the expectations of the assignment?	What does the student's work demonstrate about his/her proficiency with the requirements of the targeted standard?
		$(2 \times 4) + (2 \times 4) + (2 \times 4) + (2 \times 4) + (1 \times 4) = 9 \times 4$. The labeling "9 groups of 4 = 36 students" demonstrates that he/she is relating the multiplication equation to the context of the problem.
Student H	Student H appears to understand the problem posed.	Student H uses arrays to show different arrangements of equal groups of 36 and represented the arrays with expressions or equations. He/she understands the concept of equal groups. In the student's last drawing, he/she does not create an array, instead he/she creates two arrays (6 × 4 and 3 × 4) with a rectangle around the two arrays. The student does not relate the mathematics to the context of the problem.

Note: For a collection of more than eight samples of student work, print this page multiple times.

After looking at student work:

1. **How did the directions and/or prompts for the assignment allow students to demonstrate the requirements of the targeted standard(s)?**
   The prompt to find out how many different ways the teacher could group the students should have led the student to produce work aligned to grade 4 expectations. Since most students did not follow that direction, they were able to demonstrate the requirements of 3.OA.A ("Represent and solve problems involving multiplication and division"). A small tweak to the question (e.g., "Show different ways the teacher can group the students") would have make the prompt clearer and better aligned to grade 3 expectations.

2. **How did the mathematical content of the assignment allow students to demonstrate the requirements of the targeted standard(s)?**
   The fact that the targeted number has many factors allows students to show a range of responses to grade 3 expectations. Many students seemed to have identified known facts that make 36 (particularly 4 x 9 and 6 x 6) which shows developing fluency, as required by 3.OA.C.7. Students who used visual models to show equal groups are demonstrating mastery with standards in the 3.OA.A. cluster.

3. **What patterns do you notice in the student work?**
   - **What did students do consistently well?**
     Most students were successful at finding more than one way of creating equal groups of 36. About half the students in the sample connected their groupings to a multiplication or division sentence.
   - **Were there any common errors?**
     Few students used the context of the word problem in their answers. No other consistent errors were found.

Feedback Summary

Lesson: ______________________________

Using the completed Instructional Practice Guide, the Lesson Plan Analysis, and Student Work Analysis, consider the aggregate strengths and considerations for the lesson. Choose relevant Beyond the Lesson questions to guide longer-term reflection.

Evidence of the Shifts and standards-aligned practice	Areas where alignment to the Shifts and standards can improve
Core Action 1: Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by college- and career-ready standards in mathematics.	
Core Action 2: Employ instructional practices that allow all students to learn the content of the lesson.	

Evidence of the Shifts and standards-aligned practice	Areas where alignment to the Shifts and standards can improve
Core Action 3: Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.	
Beyond the Lesson ***Choose relevant Beyond the Lesson questions to guide longer-term reflection.***	

Implications and Next Steps

Feedback Summary

Lesson: Grade 3 – Making Equal Groups

Using the completed Instructional Practice Guide, the Lesson Plan Analysis, and Student Work Analysis, consider the aggregate strengths and considerations for the lesson. Choose relevant Beyond the Lesson questions to guide longer-term reflection.

Note for Facilitator: *The italicized statements can be used for group discussions, as a basis for developing questions for a coaching conversation with the teacher, or for participants to take a deeper dive into adapting the lesson and deepening their understanding of mathematics and the Shifts required by college- and career-ready standards.*

Evidence of the Shifts and standards-aligned practice	Areas where alignment to the Shifts and standards can improve
Core Action 1: Ensure the work of the enacted lesson reflects the Focus, Coherence, and Rigor required by college- and career-ready standards in mathematics.	
• The mathematical work that the students engaged in was on grade level and met the intent of the 3.OA.A cluster. • 3.OA.A requires students to build conceptual understanding. (Represent and solve problems involving multiplication and division), this is evident in the lesson plan, student work samples, and in the discussion students are having about the problem and how to solve and represent it. • Students were revisiting concepts of multiplication and division in order to deepen their understanding of the operations and refine their visual models and strategies for solving division problems.	• The lesson would have been more strongly aligned to grade 3 if the task presented didn't require students to find all factor pairs of 36, which is a grade 4 expectation. *What changes could be made to the task that would more closely align it to grade 3?* • The content of the lesson allowed for a stronger connection to grade 3 fluency expectations while discussing some of the solutions. This would have helped support students developing fluency based on conceptual understanding. *What questions could the teacher pose to reinforce grade 3 fluency expectations?*
Core Action 2: Employ instructional practices that allow all students to learn the content of the lesson.	
• The teacher plans for and follows through on checking for understanding by walking around the room to observe student thinking and problem solving. • The teacher strategically chooses which students will share their work with the full class in order to highlight the variety of visual models students used to represent and solve the problem.	• The specific mathematical purpose of the lesson is not clear from the lesson plan, task, or instruction. There were many ideas about multiplication and division that came up, but it wasn't clear which one the teacher thought was most important. At times, it felt like the focus was that one solution can be represented in a variety of ways and at other times, the focus seemed to be how to figure out different ways 36 can be divided into equal groups. *What adjustments could be made to the lesson plan to help clarify the purpose of the lesson?*

<table>
<tr><th>Evidence of the Shifts and standards-aligned practice</th><th>Areas where alignment to the Shifts and standards can improve</th></tr>
<tr><td colspan="2">Core Action 3: Provide all students with opportunities to exhibit mathematical practices while engaging with the content of the lesson.</td></tr>
<tr><td>• The lesson focuses on a single task and students are fully engaged in doing grade-level mathematics throughout the lesson.
• The teacher prompts students to share their reasoning throughout the lesson during one-on-one conversations, partner discussions, and when sharing with the whole class.
• The teacher intentionally plans a full class discussion where students will ask questions of each other to support their understanding and where they can offer extensions to a solution/strategy.</td><td>• It was not clear whether students received feedback on their solutions or representations or that they are being asked to revise their work. Choose two student work samples and develop follow-up questions that would allow students to revise or extend their work.
• Although the task was a word problem, there was little evidence of the students or teacher talking about the context. The student work and classroom discussion showed that most students were not thinking about context; some students wrote equations that didn't consistently match their visual models. Discuss the difference between 3 x 12 and 12 x 3 in the context of the problem and more generally as students extend their understanding of multiplication across the number system.</td></tr>
<tr><td colspan="2">Beyond the Lesson
Choose relevant Beyond the Lesson questions to guide longer-term reflection.</td></tr>
<tr><td colspan="2">• Summarize how this lesson fits within the unit. Describe how the other lessons and tasks in this unit are intentionally sequenced to help students develop increasingly sophisticated understanding, skills, and practices.

• Which of the three aspects of rigor (conceptual understanding, procedural skill and fluency, and application) are attended to within this unit? If more than one aspect is attended to, when in the unit are they attended to individually, and when are students using them together?

• In what ways have your students made progress towards mastering the grade/course-level content standards? How are you monitoring and tracking their achievement of the standards? What work still needs to be done to ensure all students achieve mastery of each standard by the end of the year?

• What off-grade/course-level standards have you taught this year and why?</td></tr>
</table>

Implications and Next Steps

Note for facilitator: Participants could use this space to reflect on questions 1 & 2, the role-specific questions, or one or more of the italicized statements from above.

1. Based on your role in the learning community, how did examining all aspects of this lesson impact your work?
2. Based on your role in the learning community, what resources and strategies could be used to encourage and support aligned instructional practice in the classroom?

Role-Specific Reflection Questions:

- **Superintendent/District Leader** – How can I direct resources to improve standards-aligned instruction in classrooms?
- **School Leader** – What building conditions must exist to support standards-aligned instruction in classrooms?
- **Coach** – How can content-based feedback help prioritize professional learning and coaching activities to support teachers with standards-aligned instruction?
- **Teacher** – Which aspects of your instructional practice provide all students with access to grade-level standards-based content and tasks? Which aspects do not?
- **Parent** – Where do you see evidence of standards-aligned instruction in your child's classroom?
- **Partner organization** – How does our organization's theory of action and activities with districts and partners support standards aligned instruction in classrooms?