Mathematical Routines

As in other subjects, math students must be able to read, write, listen, speak, and discuss the subject at hand. Often, these multimodal ways of learning and using math skills are given too little attention in curricular materials, and teachers may want to supplement with classroom activities that provide opportunities for students to use language to discuss the math content they're learning.

The routines below are designed to support a variety of language-focused skill growth: from reinforcing mathematical terminology to scaffolding conversations to providing opportunities for students to deepen their conceptual understanding by describing their work.

These routines, done regularly, can benefit *all* students, though they are particularly supportive of English Language Learners or those struggling with the linguistic components of math. The routines below are from the Understanding Language/Stanford Center for Assessment, Learning, and Equity's Principles for the Design of Mathematics Curricula: Promoting Language and Content Development and the website Fostering Math Practices. The descriptions below come directly from these sources and more detailed descriptions, step-by-step guidance, examples, and applicable classroom handouts can be found on these websites.

Mathematical Language Routines

A 'math language routine' refers to a structured but adaptable format for amplifying, assessing, and developing students' language. More information and examples of each of these routines can be found here.

MATHEMATICAL LANGUAGE ROUTINE 1: STRONGER AND CLEARER EACH TIME

Purpose: To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output (Zwiers, 2014).

This routine provides a purpose for student conversation as well as fortifies output. The main idea is to have students think or write individually about a response, use a structured pairing strategy to have multiple opportunities to refine and clarify the response through conversation, and then finally revise their original written response. Throughout this process, students should be pressed for details, and encouraged to press each other for details. Subsequent drafts should show evidence of incorporating or addressing new ideas or language. They should also show evidence of refinement in precision, communication, expression, examples, and/or reasoning about mathematical concepts.

MATHEMATICAL LANGUAGE ROUTINE 2: COLLECT AND DISPLAY

Purpose: To capture students' oral words and phrases into a stable, collective reference.

The intent of this routine is to stabilize the fleeting language that students use in order for their own output to be used as a reference in developing their mathematical language. The teacher listens for, and scribes, the language students use during partner, small group, or whole class discussions using written words, diagrams and pictures. This collected output can be organized, revoiced, or explicitly connected to other language in a display that all students can

refer to, build on, or make connections with during future discussion or writing. Throughout the course of a unit, teachers can reference the displayed language as a model, update and revise the display as student language changes, and make bridges between student language and new disciplinary language. This routine provides feedback for students in a way that increases sense-making while simultaneously supporting metaawareness of language.

MATHEMATICAL LANGUAGE ROUTINE 3: CRITIQUE, CORRECT, AND CLARIFY

Purpose: To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop.

The intent is to prompt student reflection with an incorrect, incomplete, or ambiguous written argument or explanation, and for students to improve upon the written work by correcting errors and clarifying meaning. Teachers can model how to effectively and respectfully critique the work of others with meta-think-alouds and press for details when necessary. This routine fortifies output and engages students in metaawareness.

MATHEMATICAL LANGUAGE ROUTINE 4: INFORMATION GAP

Purpose: To create a need for students to communicate (Gibbons, 2002).

This routine allows teachers to facilitate meaningful interactions by giving partners or team members different pieces of necessary information that must be used together to solve a problem or play a game. With an information gap, students need to orally (and/or visually) share their ideas and information in order to bridge the gap and accomplish something that they could not have done alone. Teachers should model how to ask for and share information, clarification, justification, and elaboration. This routine cultivates conversation.

MATHEMATICAL LANGUAGE ROUTINE 5: CO-CRAFT QUESTIONS AND PROBLEMS

Purpose: To allow students to get inside of a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms can represent different situations.

Through this routine, students are able to use conversation skills to generate, choose (argue for the best one), and improve questions, problems, and situations as well as develop meta-awareness of the language used in mathematical questions and problems. Teachers should push for clarity and revoice oral responses as necessary.

MATHEMATICAL LANGUAGE ROUTINE 6: THREE READS

Purpose: To ensure that students know what they are being asked to do, create opportunities for students to reflect on the ways mathematical questions are presented, and equip students with tools used to negotiate meaning (Kelemanik, Lucenta & Creighton, 2016).

This routine supports reading comprehension, sense-making, and meta-awareness of mathematical language. It also supports negotiating information in a text with a partner in mathematical conversation.

MATHEMATICAL LANGUAGE ROUTINE 7: COMPARE AND CONNECT

Purpose: To foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language.

Students should be prompted to reflect on and linguistically respond to these comparisons (e.g., exploring why or when one might do/say something a certain way, identifying and explaining correspondences between different mathematical representations or methods, wondering how an idea compares or connects to other ideas and/or language.) Teachers should model thinking out loud about these questions. This routine supports meta-cognitive and meta-linguistic awareness, and also supports mathematical conversation.

MATHEMATICAL LANGUAGE ROUTINE 8: DISCUSSION SUPPORTS

Purpose: To support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies (Chapin, O'Connor, & Anderson, 2009).

The examples provided can be combined and used together with any of the other routines. They include multi-modal strategies for helping students make sense of complex language, ideas, and classroom communication. The examples can be used to invite and incentivize more student participation, conversation, and meta-awareness of language. Eventually, as teachers continue to model, students should begin using these strategies themselves to prompt each other to engage more deeply in discussions.

Instructional Routines

Instructional Routines are specific and repeatable classroom structures that enable all students to engage more fully in learning opportunities that develop their mathematical thinking and reasoning. More information on each of these routines can be found here.

CONTEMPLATE THEN CALCULATE

Contemplate then Calculate is an instructional routine designed to shift attention away from mindless calculations and toward necessary structural interpretations of mathematics. This routine fosters structural thinking, MP 7. Additional resources can be downloaded free-of-cost, here. An example of this routine, applied to a Student Achievement Partners' math task, can be found here:

- Looking For and Making Use of Structure – Quadratic Equations 1 A-REI.B.4

CAPTURING QUANTITIES

Capturing Quantities is an instructional routine designed to focus students' attention on important quantities and relationships in problem situations. The goal of the routine is to develop students' ability to reason abstractly and quantitatively, MP2. Additional resources can be downloaded free-of-cost, here. Examples of this routine, applied to Student Achievement Partners' math tasks, can be found here:

- Banana Pudding 5.NF.B.7

- Sharing Chocolate 5.NF.A, 5.NF.B.3, and 4.NF.B.3d

CONNECTING REPRESENTATIONS

Connecting Representations is an instructional routine that positions students to think structurally as they connect two representations by articulating the underlying mathematics. An essential goal of this routine is expanding students' repertoire of structural noticings, MP7. Additional resources can be downloaded free-of-cost, here. Examples of this routine, applied to Student Achievement Partners' math tasks, can be found here:

- Delivering the Mail 8.F.B.4
- Profit of a Company A-SSE.B.3

RECOGNIZING REPETITION

Recognizing Repetition is an instructional routine that supports the difficult road to generalizing problem situations. Students enlist multiple modalities while they attend to the repetition in their counting, calculating, and constructing processes. In doing so, they leverage their repeated reasoning to make abstract generalizations, MP8. Additional resources can be downloaded free-of-cost, here.

THE 3 READS

The 3 Reads instructional routine is designed to develop students' ability to make sense of problems by deconstructing the process of reading mathematical situations. Over time, students will internalize this process, thereby creating a heuristic for reading and making sense of mathematical story problems, MP1. Additional resources can be downloaded free-of-cost, here. Examples of this routine, applied to Student Achievement Partners' math tasks, can be found here:

- How Many Teams Part One 4.OA.A, 4.NBT.B, 4.OA.A.3, 4.NBT.B.6
- How Many Teams Part Two 4.OA.A, 4.NBT.B, 4.OA.A.3, 4.NBT.B.6
- Box of Clay 5.MD.C
- Delivering the Mail 8.F.B.4

DECIDE AND DEFEND

Decide and Defend is an instructional routine in which students make sense of another's line of mathematical reasoning, decide if they agree with that reasoning, then draft an argument defending their decision. The routine fosters MP3, construct viable arguments and critique the reasoning of others. Additional resources can be downloaded free-of-cost, here. Examples of this routine, applied to Student Achievement Partners' math tasks, can be found here:

- Three Composing/Decomposing Problems (Jose) 2.NBT.A
- Fraction Comparisons with Pictures 3.NF.A.3d
- Cup of Rice 6.NS.A.1, 5.NF.B.7