Understanding a Fraction as a Number

3.NF.A Conceptual Understanding Mini-Assessment by Student Achievement Partners

OVERVIEW

This mini-assessment is designed to illustrate how the work of 3.NF.A expands students' understanding of the number system by introducing fractions as an extension of earlier work with whole numbers and as a subset of the rational number system, which is more fully introduced in grade 6. This mini-assessment is designed for teachers to use either in the classroom, for self-learning, or in professional development settings to:

- **Evaluate** students' understanding of aspects of 3.NF.A before or after teaching this material;
- **Gain** knowledge about assessing conceptual understanding of fractions;
- **Illustrate** CCR-aligned assessment problems;
- **Illustrate** best practices for writing tasks that allow access for all learners; and
- **Support** mathematical language acquisition by offering specific guidance.

MAKING THE SHIFTS

The 3.NF domain has one cluster: Develop an understanding of fractions as numbers. Therefore, the domain emphasizes conceptual understanding more than procedural skill and application. Using Achieve's cognitive complexity framework, this assessment was designed to measure conceptual complexity at Levels 1 and 2, as described below.

The Levels of Complexity

	Level 1	Level 2	Level 3
Conceptual Complexity	Solving the problem requires students to recall or recognize a grade-level concept. The student does not need to relate concepts or demonstrate a line of reasoning.	Students may need to relate multiple grade-level concepts of different types, create multiple representations or solutions, or connect concepts with procedures or strategies. The student must do some reasoning, but may not need to demonstrate a line of reasoning.	Solving the problem requires students to relate multiple grade-level concepts and to evidence reasoning, planning, analysis, judgment, and/or creative thought OR work with a sophisticated (non-typical) line of reasoning.

Since grade 3 is the first year where fractions are heavily emphasized, this assessment was designed to have a balance of conceptual complexity at Levels 1 and 2, with some items measuring the basic understandings articulated in the standards and others reaching a bit higher by asking students to do some reasoning with or relating of the concepts.

Alignment Summary & Complexity Analysis for the Assessment

Item	Standard(s)	Cognitive Complexity Analysis	Item	Standard(s)	Cognitive Complexity Analysis
1	3.NF.A.2.a	Conceptual 1	7	3.NF.A.3.d	Conceptual 2
2	3.NF.A.2.b	Conceptual 1	8	3.NF.A.3.a	Conceptual 1
3	3.NF.A.1	Conceptual 1	9	3.NF.A.3	Conceptual 2
4	3.NF.A.3.c	Conceptual 2	10	3.NF.A.3.b	Conceptual 1, Procedural 2
5	3.NF.A.2	Conceptual 1	11	3.NF.A.3.c	Conceptual 1
6	3.NF.A.3.d	Conceptual 2	12	3.NF.A.3.d	Conceptual 2

A CLOSER LOOK

A couple of key decisions informed the work on this assessment:

- Since there is no requirement in the grade 3.NF standards for solving real-world problems, this assessment does not contain any problems with real-world context to ensure that the evidence elicited by the items clearly reflects the intended expectations of the standards.
- To ensure that the body of evidence is targeting the key understanding of "fractions as numbers," the assessment has a heavier balance of items that represent fractions on a number line and fewer that use area model representations.

SUPPORT FOR ENGLISH LANGUAGE LEARNERS

This assessment was designed to include specific features that support access for all students and align to best practice for English Language Learner (ELL) instruction and assessment. Go here to learn more about the research behind these supports. Features that support access in this mini-assessment include:

- Tasks that allow for multi-modal representations, which can deepen understanding of the mathematics and make it easier for students, especially ELLs, to give mathematical explanations.
- Tasks that avoid unnecessarily complex language to allow students, especially ELLs, to access and demonstrate what they know about the mathematics of the assessment.

Prior to this mini-assessment, ensure students have had ample opportunities in instruction to read, write, speak, listen for, and understand the mathematical concepts that are represented by the following terms and concepts:

- **Number line diagram**
- **Whole**
- **Compare**
- **Greater value**
- **Equivalent to**
- **Fractions**

Students should engage with these terms and concepts in the context of mathematical learning, not as a separate vocabulary study. Students should have access to multi-modal representations of these terms and concepts, including: pictures, diagrams, written explanations, gestures, and sharing of non-examples. These representations will encourage precise language, while prioritizing students' articulation of concepts. These terms and concepts should be reinforced in teacher instruction, classroom discussion, and student work (for example, through engagement in mathematical routines).

ELLs may need support with the following Tier 2 words found in this mini-assessment:

- **Circle**
- **Explain**
- **Point**
- **Place**
- **Label**

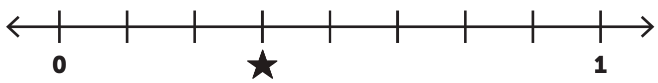
In preparation for giving this mini-assessment, teachers should strive to use these words in context so they become familiar to students. It will be important to offer synonyms, rephrasing, visual cues, and modeling of what these words mean in the specific contexts represented in the items in this mini-assessment. Additionally, teachers may offer students the use of a student-friendly dictionary or visual glossary to ensure they understand what is being asked of them in each item.

UNDERSTANDING A FRACTION AS A NUMBER MINI-ASSESSMENT

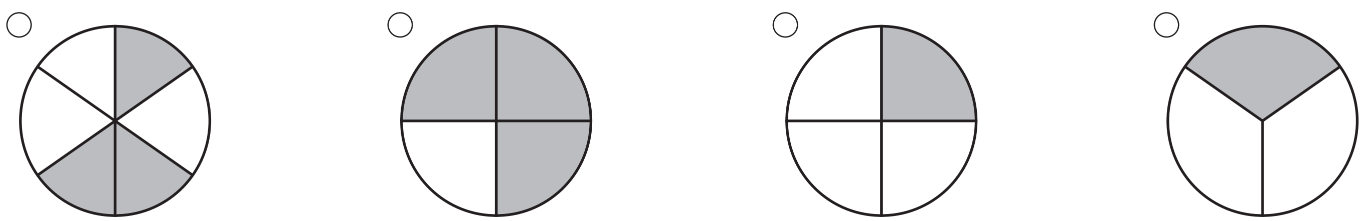
Name: ______________________________ Date: ______________________________

1. The number line diagram shows the numbers 0 and 1. Place a point at ⅓ on this number line diagram.

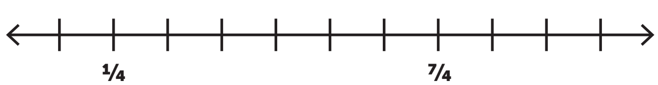
2. What number is located at the star shown on the number line diagram? __________



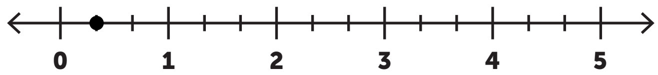
3. Which model shows ¾ of a whole circle shaded?

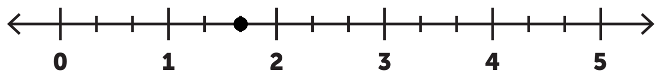


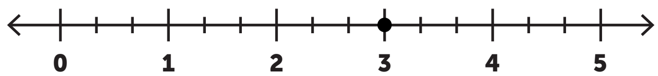
4. Place points at the numbers 1 and 2 on the number line diagram shown. Label each point.

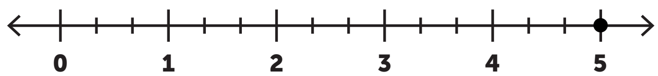


5. Which number line diagram shows the correct location of the number $\frac{5}{3}$?

○ 

○ 

○ 

○ 

6. (a) Write a statement using $<$, $>$, or $=$ to compare $\frac{2}{8}$ and $\frac{2}{6}$.

____

(b) Explain why the statement you wrote is true using pictures, numbers, and/or words.

7. For each pair of numbers, circle the number that has the **larger** value.

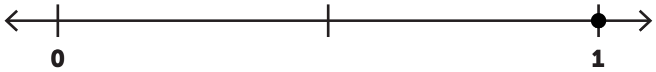
Circle the larger number: $\frac{3}{8}$ OR $\frac{5}{8}$

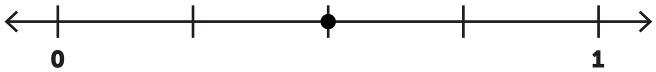
Circle the larger number: $\frac{7}{4}$ OR $\frac{5}{4}$

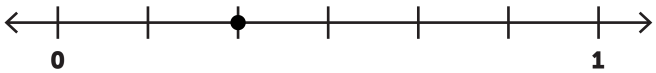
Circle the larger number: $\frac{1}{4}$ OR $\frac{1}{8}$

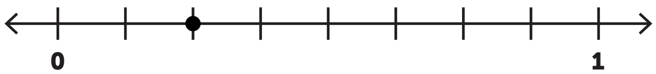
Circle the larger number: $\frac{9}{8}$ OR 1

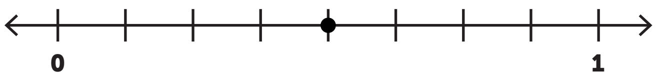
8. (a) Which number line diagram has a point located at $\frac{1}{4}$?

○ 

○ 

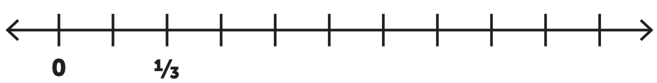
○ 

○ 

○ 

(b) Explain how you know that the point is located at $\frac{1}{4}$.

9. Place points at the numbers $\frac{2}{3}$ and $\frac{7}{6}$ on the number line diagram shown. Label each point.

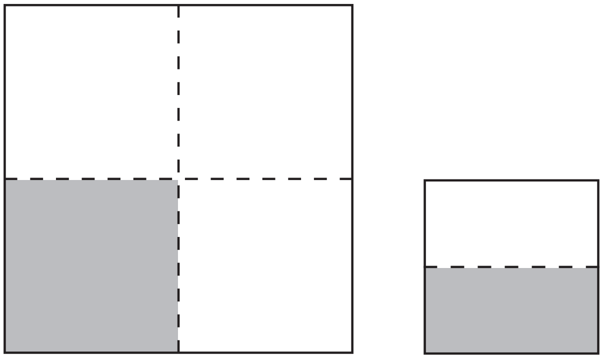


10. Write two fractions that are equivalent to $\frac{3}{6}$. ________ ________

11. Which **three** fractions are equivalent to whole numbers?

○ $\frac{8}{4}$ ○ $\frac{5}{4}$ ○ $\frac{3}{2}$ ○ $\frac{1}{2}$ ○ $\frac{3}{1}$ ○ $\frac{6}{6}$

12. Bryce drew this picture:



Then, he said, "This shows that ¼ is greater than ½".

Bryce made a mistake.

(a) Explain Bryce's mistake.

(b) Draw a picture that you could use to help Bryce correctly compare ¼ and ½.

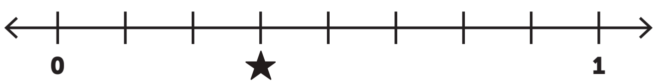
ANSWER KEY AND ANALYSIS

1. The number line diagram shows the numbers 0 and 1. Place a point at ⅓ on this number line diagram.

Answer Key: Student places a point approximately ⅓ the distance from 0 to 1.

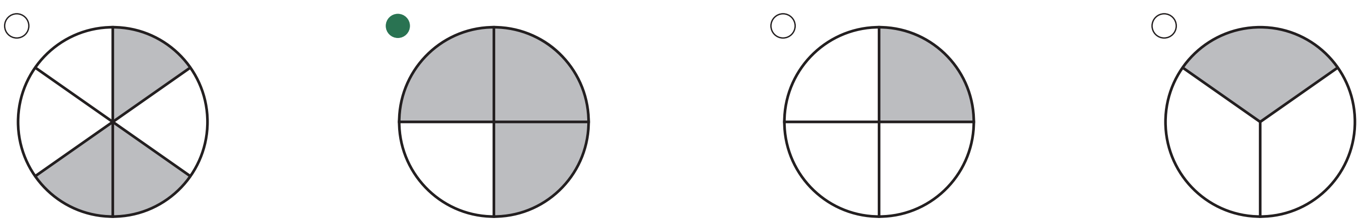
> 3.NF.A.2.a requires students to represent a fraction $^{1}/_{b}$ on a number line diagram by defining the interval from 0 to 1 as a whole and partitioning it into *b* equal parts. Students should have plenty of opportunities to do their own partitioning and connect this to the naming of fractions on the number line.

2. What number is located at the star shown on the number line diagram? **Answer Key: ⅜**



> 3.NF.A.2.b requires students to recognize that a fraction $^{a}/_{b}$ is located in a position defined by marking off *a* lengths of $^{1}/_{b}$ equal parts.

3. Which model shows ¾ of a whole circle shaded?



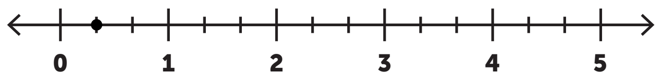
> 3.NF.A.1 requires students to understand a fraction $^{a}/_{b}$ as the quantity formed by a parts of size $^{1}/_{b}$. This item is the one on the assessment most closely tied to students' work in grade 2 (2.G.A.3) where they partitioned circles into halves, thirds, and fourths. As noted in the opening, the area model is less represented on this assessment due to the focus of the grade 3 work on understanding a fraction as a number.

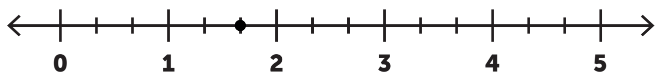
4. Place points at the numbers 1 and 2 on the number line diagram shown. Label each point.

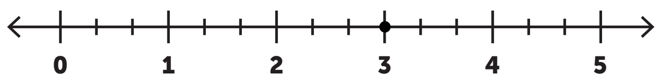
Answer Key: Student places two points and labels them 1 and 2 as above.

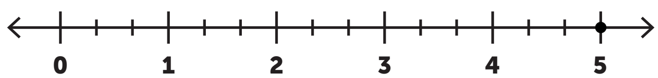
> This item is aligned to 3.NF.A.3.c, which requires students to express whole numbers as fractions, but sits at a higher level of complexity than typical items that assess this standard. The number line does not show the number zero. This is an intentional decision to position fractions as an extension of students' earlier work with whole numbers where they were often asked to determine "missing whole numbers" in a sequence or on a number line. See [Choral Counting: A Lever to Develop Fraction Understanding] for additional insight into this alignment.

5. Which number line diagram shows the correct location of the number $5/3$?

- ○ 

- ● 

- ○ 

- ○ 

> Similar to #2, this item seeks evidence of students' understanding of a fraction $^{a}/_{b}$, but this item shows the importance of fractions greater than one as attending to the full meaning of the grade 3.NF standards. All four number lines show thirds, so students should recognize the second answer choice as the one with a point that is 5 one-third lengths away from the number zero. Connecting to the unit fraction reflects an important shift in the standards that allows for students to understand fractions greater than one in the same manner as fractions less than one.

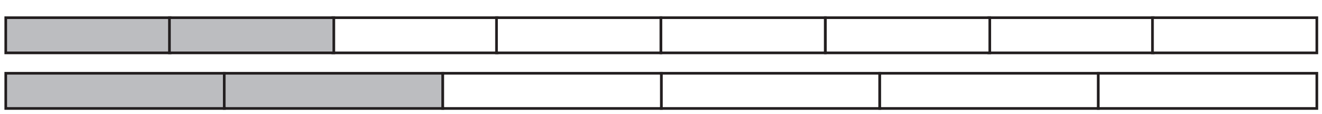
6. (a) Write a statement using <, >, or = to compare $2/8$ and $2/6$.

Answer Key: $2/8 < 2/6$ OR **$2/6 > 2/8$**

(b) Explain why the statement you wrote is true using pictures, numbers, and/or words.

Sample response:
Since the number of pieces (the numerators) are the same, I compared the size of the pieces (the denominators). A one-eighth size piece is less than a one-sixth size piece.

OR



$2/8$ covers a smaller area than $2/6$. The more pieces you cut your whole into, the smaller the size of each piece. Therefore, eighths are smaller than sixths.

3.NF.A.3.d requires students to compare two fractions with the same numerator or the same denominator by reasoning about their size, and students must also justify their conclusions. This problem allows the students to demonstrate this understanding by explaining the comparison they've written, giving greater insight into different ways that students think about fraction comparisons.

7. For each pair of numbers, circle the number that has the **larger** value.

Circle the larger number: $\frac{3}{8}$ OR $\frac{5}{8}$

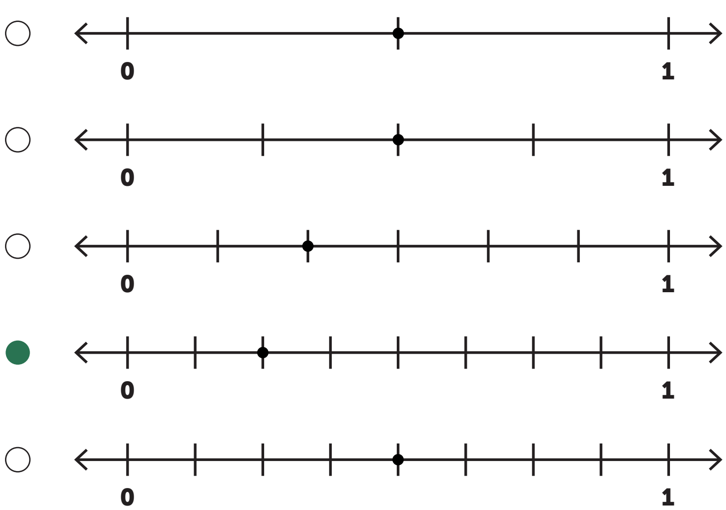
Circle the larger number: $\frac{7}{4}$ OR $\frac{5}{4}$

Circle the larger number: $\frac{1}{4}$ OR $\frac{1}{8}$

Circle the larger number: $\frac{9}{8}$ OR 1

3.NF.A.3.d requires students to compare two fractions with the same numerator or the same denominator by reasoning about their size. This problem is similar to the problem above, but the student is not required here to use the symbol to compare. This was an intentional choice in the design of the assessment to allow a pure measure of the student's ability to reason about the value of the numbers without the added complexity of mathematical notation.

8. (a) Which number line diagram has a point located at $\frac{1}{4}$?



(b) Explain how you know that the point is located at $\frac{1}{4}$.

Answer Key: I know that $\frac{1}{4} = \frac{2}{8}$. The number line I chose shows eighths and has a point at $\frac{2}{8}$ which is also $\frac{1}{4}$.

3.NF.A.3.a requires students to understand two fractions as equivalent (equal) if they are the same size or represent the same point on a number line. This question requires students to demonstrate understanding of how fraction equivalence is related to two numbers being at the same position on the number line.

9. Place points at the numbers $2/3$ and $7/6$ on the number line diagram shown. Label each point.

Answer Key: Student places two points and labels them $2/3$ and $7/6$ as above.

This item is aligned to the parent standard 3.NF.A.3, instead of one of its parts, because it integrates knowledge from across multiple standards, and students may approach it differently. For example, a student may first want to label $1/3$ as also being $2/6$ since it is at the second mark from the number zero. Another student may start by labeling $2/3$ and $3/3$ using their understanding of counting off distances of $1/3$ to locate other fractions with denominator 3.

10. Write two fractions that are equivalent to $3/6$. __________ __________

Answer Key: Accept all correct answers.
Possible answers: $1/2$ $2/4$ $4/8$ $6/12$

NOTE: While $3/6$, $3/6$ is technically correct, students should be encouraged to come up with different answers if they write this. Offering additional explanation in order to prevent this response would have increased the linguistic complexity of the problem unnecessarily for an issue that is unlikely to occur often.

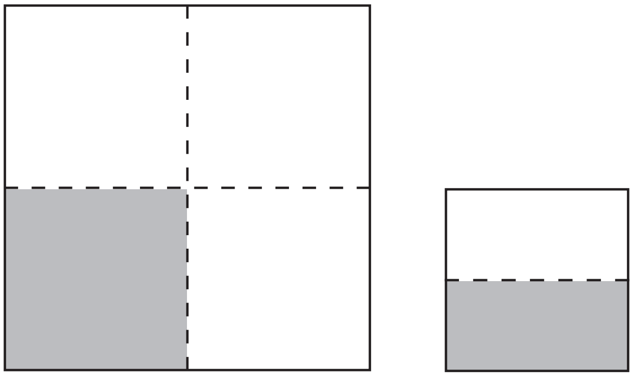
This is the only item on the assessment where some students may use some level of procedural skill to generate equivalent fractions. However, because the work of grade 3 is still heavily focused on understanding equivalence, this item is coded as both procedural and conceptual. It is designed to measure 3.NF.A.3.b, which asks students to generate simple equivalent fractions.

11. Which **<u>three</u>** fractions are equivalent to whole numbers?

- ● $8/4$
- ○ $5/4$
- ○ $3/2$
- ○ $1/2$
- ● $3/1$
- ● $6/6$

While #4 (also aligned to 3.NF.A.3.c) was of higher complexity, this problem is a bit more straightforward in measuring students' ability to recognize fractions that are equivalent to whole numbers.

12. Bryce drew this picture:[1]



Then, he said, "This shows that ¼ is greater than ½".

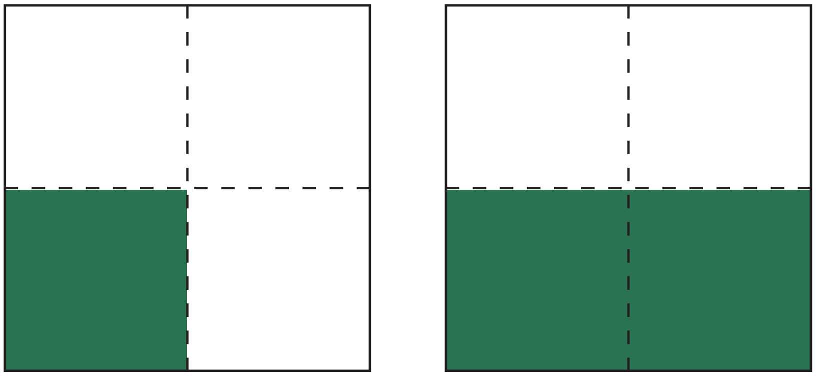
Bryce made a mistake.

(a) Explain Bryce's mistake.

Answer Key: While it is true that the shaded region on the right has a smaller area than the shaded region on the left, the picture doesn't help us to compare ¼ and ½. The reason is that the two wholes have different areas, and are thus different sizes.

In order to correctly compare these two fractions, we need to draw a picture of them with the same-sized whole.

(b) Draw a picture that you could use to help Bryce correctly compare ¼ and ½.



Answer Key: The picture above shows the same whole divided into 4 pieces on the left and 2 pieces on the right. ¼ is shaded on the left and ½ is shaded on the right. Because these two fractions refer to the same-sized whole, it is easy to see that ½ is greater than ¼.

> 3.NF.A.3.d requires a student to compare two fractions with the same numerator or the same denominator by reasoning about their size, and to recognize that comparisons are valid only when the two fractions refer to the same whole.
>
> This problem is meant to address a common error that students make, namely, that they represent fractions with different wholes when they need to compare them. It is important that students understand when you compare fractions, the wholes have to be the same.

[1] This question and its explanation come from Illustrative Mathematics: https://tasks.illustrativemathematics.org/content-standards/3/NF/A/3/tasks/880