

REPORT IN BRIEF

Preparation for Success in Algebra: Exploring Math Education Relationships by Analyzing Large Data Sets (EMERALDS)

$$\frac{2^5}{2^3} = 4$$

$$3(4^2 + 1) =$$

301

STUDENT
ACHIEVEMENT
PARTNERS

Introduction

Algebra is the language of higher-level mathematics, a passport for expanding postsecondary opportunities and a tool for confidently navigating the quantitative demands of daily life (NMAP; National Mathematics Advisory Panel, 2008). Improving students' understanding of algebra has been a long-term educational priority in the United States; however, achieving this goal has been elusive (Stein et al., 2011), especially for students who have been historically underrepresented in STEM fields.

The Exploring Math Education Relationships by Analyzing Large Data Sets (EMERALDS) study is an attempt to use existing data sets to identify specific CCSS procedural, conceptual, and problem-solving competencies in earlier grades that provide the most critical foundation for success in algebraic areas in later grades. This endeavor is an effort to inform policy that will improve outcomes and better support all students, but especially those who are Black, Latino, English learner-designated, experiencing poverty, and/or female and have been historically underserved.

This EMERALDS study used anonymized data to try to glean insights, though not for purposes of looking critically at students or teachers, but rather for the purposes of looking critically at the educational system as a whole and school mathematics itself.

Methodology

EMERALDS involved large-scale, four-year longitudinal data sets from 1,108,844 students across the states of Idaho (42,474 students), Washington (216,595 students), and California (849,775 students). The state student measure used was the state-administered Smarter Balanced Assessment Consortium (SBAC) assessment, which is based on Common Core State Standards for Mathematics (hereafter CCSS).

Content experts identified core Prealgebra knowledge domains and Algebra content areas with the goal of establishing clusters—groups of related items—for use in the analyses. The foci of item clusters is on major arithmetic-based elementary-grade topics, such as fractions concepts, that are thought to be foundational for algebra learning in later grades. Competencies in other areas, such as measurement and geometry, were also examined, to determine if success with these major elementary topics was more predictive of future success in algebra than the core elementary school topics.

Student performance in earlier grades was directly linked to their algebra performance in later grades. We included three cohorts of students from Idaho and Washington and created predictors based on performance in third to fifth grades, inclusive, and outcomes based on performance in sixth to eighth grades, inclusive. There were more students in California; hence, we were able to create predictors and outcomes using only two cohorts, resulting in combined fourth- and fifth-grade predictors and seventh- and eighth-grade outcomes (see also Weeks & Baron, 2021). The design of the SBAC computer-adaptive assessment makes it well suited to providing an overall estimate of mathematics competencies, but our analyses suggest that measures such as this are not as well suited to identifying fine-grained subsets of competencies such as those explored in this study. Given the challenges of the data sets, we tried two approaches for the analysis to try to identify relationships between earlier grades content and algebra outcomes: analyzing Item Response Theory (IRT) residuals and IRT subscores. The second approach was more fruitful, and was the method used to generate the study's findings.

The study also used more detailed assessments of Florida middle school students' engagement with pre-algebraic and algebraic material in the computer adaptive tutor MATHia, as related to their Algebra I end-of-course performance (1,304 students). Carnegie Learning's MATHia, (formerly known as Cognitive Tutor; Ritter et al., 2007) is part of Carnegie Learning's blended curriculum for middle school and high school mathematics. In addition to the MATHia data, student performance data was also available for the Florida Standards Assessment (FSA) that includes ELA in grades 3–10 and mathematics in grades 3–8 and for the required End-of-Course (EOC) assessments for Algebra 1 and Geometry. The core focus of the regression analysis was on the Mathia workspaces attempted and completed by students for whom we also had Algebra I EOC scores.

Findings

Overall mathematical performance in elementary grades is a substantive predictor of later Algebra outcomes. This finding is true irrespective of student gender, ethnicity, race, disability status, English learner designation, eligibility for reduced or free lunches, students' prior English language arts competencies, or school/district.

This finding does not imply that students should be denied algebra in middle grades contingent on their level of arithmetic competency, but rather that educational systems should be aware of the stakes of early grades' learning for what comes next.

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Using the IRT subscore approach, we found that overall mathematical performance in grades 3-5 is a substantive predictor of later overall Algebra outcomes. The effect of elementary grades' overall mathematics competence for prediction of the overall Algebra outcome (b coefficient) was significant and very similar across all three state data sets: 0.454 in the California data, 0.533 in the Idaho data, and 0.508 in the Washington data (see the last row of the table).

To put these results in perspective, if a school-level intervention improved student scores at these levels, about 70% of the students would score above the pre-intervention 50th percentile.

Comparison of Findings Across State Data: IRT Subscores for the Core Statistical Model

	California	Idaho	Washington
Predictors	Overall Algebra Beta	Overall Algebra Beta	Overall Algebra Beta
A1a - Whole Numbers	0.037	0.080	0.046
A1b - Fractions	0.023	-0.007	0.018
A1c - Decimals	0.013	0.011	0.012
A2a - Basic Problem Solving: Whole Numbers	NA	-0.019	0.012
A2b - Basic Problem Solving: Fractions	0.017	0.037	0.026
A3a – Complex Problem Solving: Whole Numbers	0.035	0.050	0.034
A3b – Complex Problem Solving: Fractions	0.053	0.024	0.039
A4 – Mathematical Reasoning and Communication	0.130	0.048	0.102
Geometry & Measurement	0.010	0.002	0.003
ELA	0.165	0.157	0.152
Math	0.454	0.533	0.508

Note: NA means the subscale was not reliable and thus not included.

IRT subscores from California were used to examine whether there were differences among groups of students. The analyses assessed whether there were differences across demographic groups in the *strength of the relation* between earlier performance in Mathematical Reasoning and Communication and overall mathematics competence and later overall Algebra scores. These analyses revealed several statistically significant effects, but these were all small and of little practical importance.

In all, strong performance in elementary mathematics CCSS, which emphasizes the concepts, procedural fluencies, and applications of arithmetic, was important for the later Algebra performance of all students. This is not to say, however, that students have had the same opportunities to learn this foundational material, but rather those who have solid skills by the end of the elementary school years (independent of demographic group) are on track for later success in Algebra. Similarly, we explored whether there were differences between schools and districts, and in California, the results indicated that districts and schools were more similar than different, once prior achievement was considered.

Taken together these findings indicate that readiness for Algebra is dependent on a solid foundation in the elementary school CCSS. The results suggest a validation of the balanced rigor framed by the CCSS for arithmetic as the foundation for Algebra. The more specific predictors of later Algebra outcomes suggested effects that cut across content topics: multi-step problems were more predictive than one-step problems, and Mathematical Reasoning and Communication added the most predictive value above and beyond the total mathematics score.

To put this result in perspective, if a school-level intervention resulted in the average student successfully completing 30 more CCSS workspaces, about 66% of the students would score above the pre-intervention 50th percentile. With successful completion of 60 workspaces, almost 80% of students would score above the pre-intervention 50th percentile.”

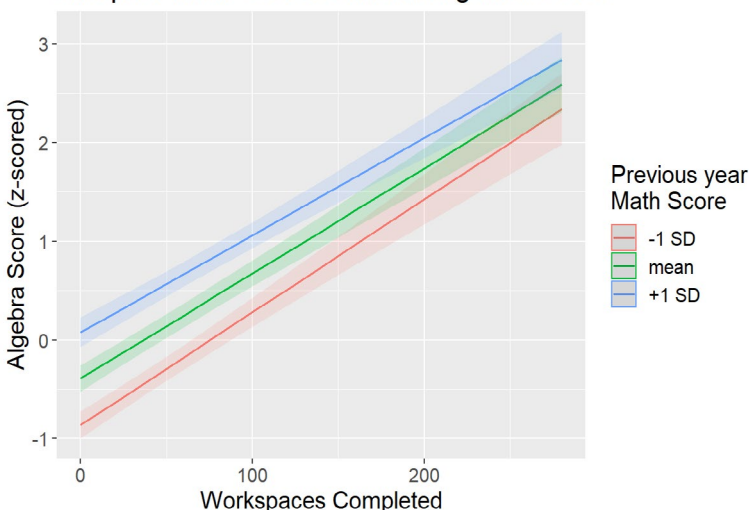
Engagement with computer adaptive tutoring with multi-step problems during the middle school years results in important gains in later Algebra I performance, controlling for prior mathematics competencies, and can help students who scored lower on prior mathematics assessments to gain more ground than their peers with higher prior mathematics scores.

Middle school students’ practice with and mastery of algebra-related content in the middle school years was also a substantive predictor of later Algebra I EOC performance. For each 30 Mathia CCSS workspaces successfully completed, there was a substantive improvement (= .406) in later Algebra I EOC performance, controlling for prior mathematics competence. To put this result in perspective, if a school-level intervention resulted in the average student successfully completing 30 more CCSS workspaces, about 66% of the students would score above the pre-intervention 50th percentile. With successful completion of 60 workspaces, almost 80% of students would score above the pre-intervention 50th percentile.

Results of Regression Predicting Algebra I EOC Exam Scores Based on Different Types of Tutor Usage Frequency

Predictor	Estimate	Std Error	t	p
(Intercept)	-1.747	0.225	-7.77	0.000
Workspaces Mastered	0.406	0.024	17.28	< .001
Workspaces Not Completed	-0.029	0.011	-2.65	0.008
Workspaces Not Mastered	-0.167	0.012	-13.45	< .001
Previous Mathematics Score	0.707	0.020	35.66	< .001
Mathematics Course Grade (grades 6 or 7)	0.232	0.027	8.57	<.001
Interaction: Mastered by Previous Mathematics Score	-0.049	0.016	-3.14	0.002

Relation between Workspace completion and prior math scores on later Algebra scores

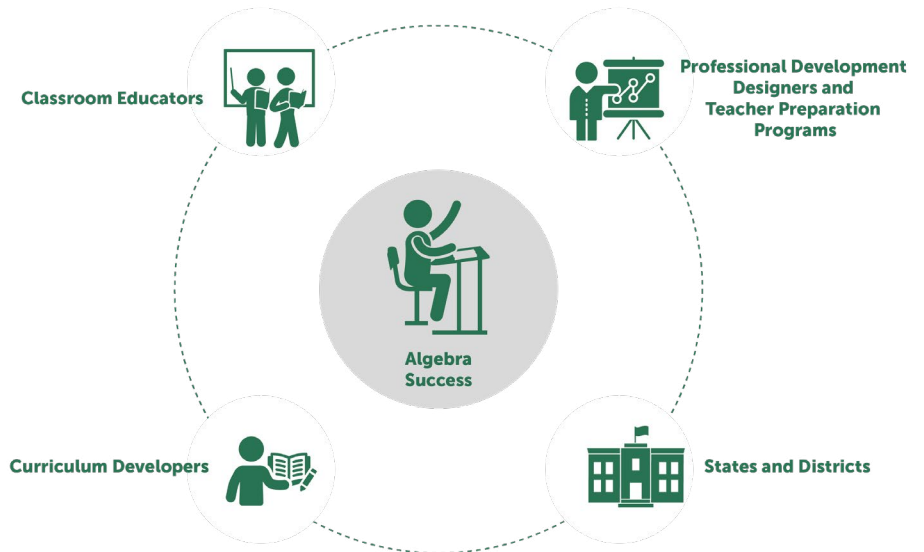


Importantly, the practice helped students who scored lower on prior mathematics assessments to gain more than their peers with higher prior mathematics scores. The interaction effect (based on parameter estimates) suggests that students with lower scores gain more than ones with higher scores by mastering additional workspaces. In other words, the difference between higher- and lower-scoring students in later Algebra I EOC scores becomes smaller with increases in the number of mastered workspaces.

Recommendations

This study provides evidence that readiness for Algebra is dependent on a solid foundation in the elementary school arithmetic-focused CCSS, irrespective of student gender, ethnicity, race, disability status, English learner designation, eligibility for reduced or free lunches, students' prior English language arts competencies, or school/district.

Further, we found preliminary evidence that engagement with computer adaptive tutoring with pre-algebra and algebra multi-step problems during the middle school years results in important gains in later Algebra I performance, controlling for prior mathematics competencies. This type of computer adaptive tutoring can help students who scored lower on prior mathematics assessments to gain more ground than their peers with higher prior mathematics scores.



The following recommendations weave together the findings of this study, research outside the study, and the discussions of the Research Advisory Committee.

S=study findings;

PR=prior research;

RAC=Research Advisory Committee discussions

Classroom-focused



Curriculum-focused



Professional learning focused



Future research



What should states and districts do to help every student leave elementary school with a solid foundation in elementary school CCSS and support middle school mathematics growth?



1. **Communicate to teachers, students, families and caregivers, and the community the importance of a strong mathematical foundation in elementary grades for later success in Algebra. The messaging must not lead to middle-grade students being denied opportunities to learn pre-algebra and algebra on the basis of their opportunities in elementary grades, since the findings in this report emphasize the progress that all students can make in middle-grade Algebra. Thus, provide educators with resources and professional learning aimed at the goal of regularly engaging students in grade-level and challenging mathematics even in cases where the educational system has not provided them an adequate mathematical foundation (Balfanz, Mac Iver & Byrnes, 2006; Baker, Gersten & Lee, 2002; Burris, Heubert & Levin, 2006; Global Family Research Project, 2017; TNT, 2021).**

S, PR, RAC



2. **Adopt an integrated, arithmetic-focused curriculum for the entire elementary grade span. Structure adoption processes to ensure the curriculum is designed to explicitly support teachers to facilitate the learning of students who have been historically marginalized by ensuring their unique identities, culture and needs are honored.** The curriculum should be coherently organized around key content threads (e.g., an understanding of numerical magnitude) that tie material together across grades in order to better prepare students for later success in Algebra. (Brown, 2007; Howard-Hamilton, 2002; Santamaria, 2009).

PR, RAC



- 3. Support student transitions from elementary to middle school and middle school to high school by maintaining coherence of the K–12 mathematics learning pathway.** For example, adopt curricular materials that build coherently across the grades, and ensure that school-based staff understand the value of instructional coherence across the grades in their school and beyond (ACT, 2008; National Research Council, 1999).

PR, RAC



- 4. Provide professional learning opportunities that help teachers develop their own strong mathematical identity and a solid understanding of the key mathematical threads of their curricular programs.**

(Feiman-Nemser, 2001; Gallagher, 2016; Schoenfeld, 2014).

PR, RAC



- 5. Consider providing students with supplemental grade-level practice for content with significant evidential support for improving Algebra performance** (e.g., fractions in upper elementary grades, Siegler et al., 2012; Algebra by Example during Algebra 1, Booth et al., 2015). The supplementals should support and coherently reinforce the tier 1 instructional materials (Gersten, Beckmann, Clarke, Foegen, Marsh, Star & Witzel, 2009).

S, PR

What should curriculum developers do to support states, districts, teachers, and students to succeed in teaching and learning a coherent mathematics curriculum?



- 6. Invest in designing materials and explicit support for teachers in order to focus on students who have been historically marginalized by ensuring their unique identities, culture, and needs are honored.** For example, consider how the curricular materials cultivate or become a barrier to cultivation of positive mathematical identities for students who are Black and/or Latino, and how they engage students learning English. This is in service to the core goal of students acquiring a solid understanding of whole number and rational number arithmetic during the elementary school years and prealgebra and early algebra in the middle school years (Leonard, Knapp & Adeleke, 2009; Peoples, Islam & Davis, 2021; Ukpokodu, 2011).

PR, RAC



- 7. Design curricular materials and programs—including supplementals—in the elementary school grades that emphasize the concepts, procedural fluencies, and applications of arithmetic as well as CCSS's practice standards for complex problem solving, communicating reasoning, and the ability to use modeling to solve real-world problems.** Curricular materials and programs should be coherently organized around key content threads and tie material together across grades. Specifically, these key threads would include number sense, that is, a developing understanding of numerical magnitudes (including fractions and later rational numbers) and the arithmetical operations that can be applied to them (Siegler & Braithwaite, 2017). Success at using this knowledge to better understand mathematical relationships as well as to apply it to problem-solving contexts, as in word problems, is a critical component of early mathematics education and preparation for later algebra.

S, RAC



- 8. Attend to the content and coherence of the curricula materials, but also their mathematical fidelity and the quality of the mathematical tasks with which students are asked to engage.** For instance, in pre-CCSS textbooks in the United States, arithmetic problems were typically presented overwhelmingly in a result-unknown format as $a + b = ?$ (e.g., $4 + 3 = ?$), an approach that results in many students inferring that the '=' sign means to operate on the numbers to the left rather than indicating the equality of the quantities to the left and right of it (McNeil et al., 2006). Textbooks must follow CCSS in this area (see, e.g., [1.OA.D.7](#)) by coherently integrating the forms of number relationships (e.g., $c = b + a$) that express number decompositions and facilitate students' understanding of the '=' as a relational construct (McNeil et al., 2011).

PR, RAC

What should designers of professional development and teacher preparation programs do to support teachers in helping historically marginalized students succeed in learning a solid foundation in elementary mathematics and have success in Algebra?



9. **Design professional learning which helps teachers develop their own strong mathematical identity in order to positively impact their teaching of mathematics** (Ball & Forzani, 2011; Thompson, 1992; Wei, Darling-Hammond, Andree, Richardson & Orphanos, 2009).

PR, RAC



10. **Design professional learning to support K–5 teachers to develop a solid understanding of the key threads of their curricular programs, specifically how knowledge at earlier grades provides the foundation for later learning and is not only a steppingstone but also is conceptually related to later material.** Professional learning should help teachers support students' unfinished learning by building on their understandings and assets to access the topic at hand, as opposed to re-teaching prior-grades material. Professional learning should also help teachers build their knowledge and ability to navigate decisions about when and how to modify the curriculum to make it stronger and more relevant for students, and not make changes that unravel the coherence and priorities of a strong curriculum (as described above). For example, improvising and skipping tasks in number and operations can cause incoherence in the curriculum leading to algebra. Teachers who are well equipped understand that working through these tasks provides additional practice on basic skills. (Darling-Hammond, Hyler & Gardner, 2017; National Mathematics Advisory Panel, 2008).

PR, RAC



11. **Require preservice teachers to take one or more courses aimed at helping teachers develop a solid mathematical understanding of fundamental mathematical concepts and the conceptual connections.** (Conference Board of the Mathematical Sciences, 2012; National Research Council, 2001).

PR, RAC

What should researchers do to support students, teachers, states, districts, curriculum developers and designers of professional development and teacher preparation programs?



12. **Explore in greater detail the core components of a strong elementary school mathematics foundation through a modified replication of this study,** using a non-computer-adaptive assessment with sufficient items in key predictor content from other states with different geographic and student demographic profiles; the study may require the inclusion of additional assessment items to assess core math areas (e.g., fractions). Such a study would benefit from close partnership with state departments of education and districts. One goal of the latter should be to better understand aspects of curricular implementation and other contextual factors for mathematics success, such as student experience.

S



13. **Follow up on the promising middle school MATHia results to understand the extent to which engagement with computer adaptive tutoring during the middle school years results in gains in later Algebra I performance.** A follow up study with larger samples as well as a greater understanding of the usage and student experience would help to verify these findings and enable a more fine-grained assessment of how computer adaptive tutoring supports mathematical development and identity, and if there are experience and usage differences across students who are Black, Latino, English learner-designated, experiencing poverty, and/or female. Of course, strong causal conclusions will also have to await randomized controlled trials.

S



14. Look inside upper elementary classrooms to learn about key curricular and instructional factors that make a difference for students who are Black, Latino, English learner-designated, experiencing poverty, and/or female and who are successful in upper elementary grades mathematics. Studying the practices of teachers of students who are Black, Latino, English learner-designated, experiencing poverty, and/or female and are succeeding in learning upper elementary mathematics can help identify key factors that can support students on the path for future success in Algebra. Observations and interviews of students and teachers would document student-teacher relationships, classroom or school environment, curriculum implementation, and instructional practices that contribute to student success.

RAC



15. Examine how targeted professional learning for upper elementary teachers may affect the performance of students. Using research-backed practices, design, deliver and evaluate the effectiveness of professional learning which integrates Mathematics Practice 1 (make sense of problems and persevere in solving them), fractions, supporting student identity, and instruction for equity. A study such as this would require the active involvement of district staff, school leaders, teachers, caregivers, and students, including but not limited to interviews and surveys, observations, student work analysis, and shared interpretation of findings.

RAC

For the full report, please visit achievethecore.org/successinalgebra.

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